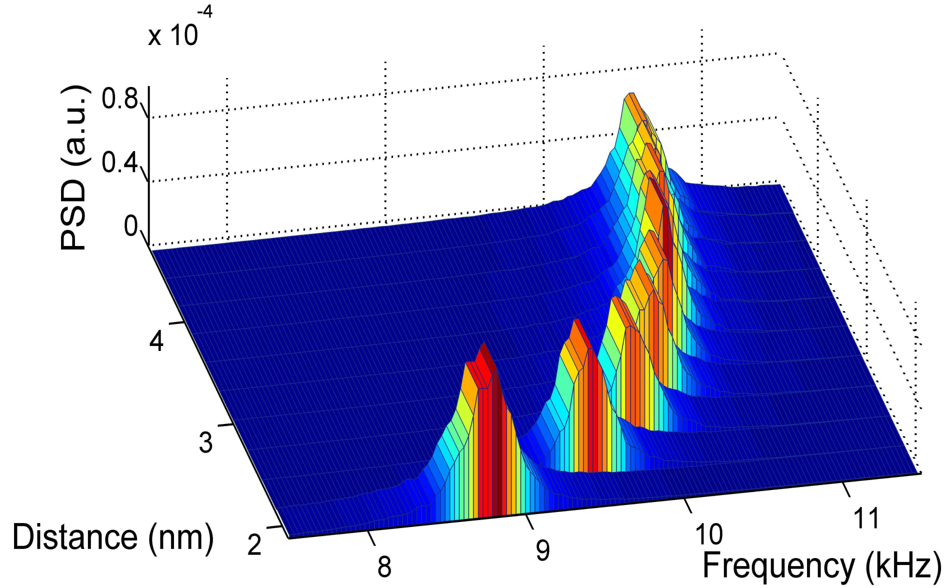
Tip-sample interaction on graphite studied in the thermal oscillations regime

Giovanna Malegori



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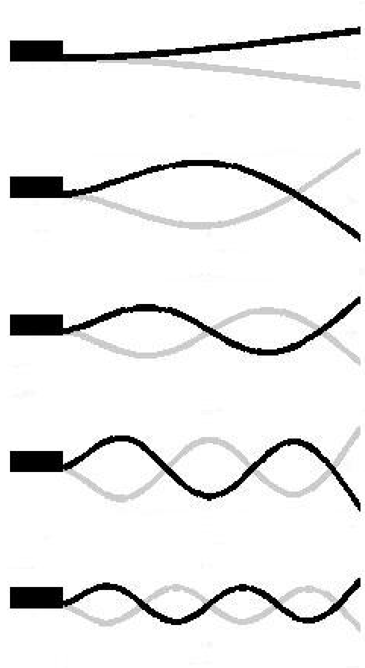
- Introduction
- Probing the tip-sample interaction
  - Frequency shift
  - Potential from Boltzmann distribution
  - Mean-square displacement from power spectral density

Thermal vibrations

AFM cantilevers display thermal flexural oscillations

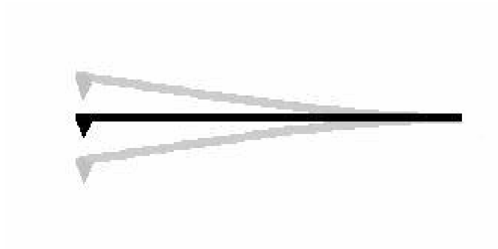
- noise source
- calibration of the cantilever spring constant *k* [*]
- **probing van der Waals tip-sample interaction**



[*] J.E.Sader *et al.*, *Rev. Sci. Ins.* **70**, 3967 (1999)

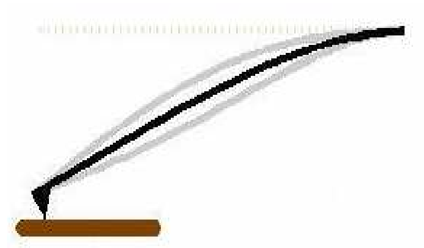
Thermal vibrations

- The free AFM cantilever oscillates due to random thermal excitations



- As the tip approaches the sample surface, the tip-sample interaction deflects the cantilever and modifies its thermal vibrations → tip-sample force gradient

- The jump-to-contact occurs due to liquid meniscus pulling the tip (humidity) [*]



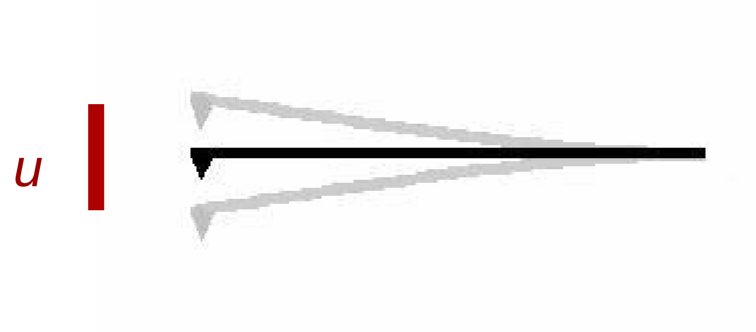
[*] M. Luna *et al. J. Phys. Chem B* **103**, 9576 (1999)

Free cantilever thermal motion →

Langevin equation

$$m^* \frac{d^2u}{dt^2} + \gamma \frac{du}{dt} + ku = F_{rand}$$

- *u* cantilever displacement
- *k* spring constant
- *m** effective mass
- γ damping
- F_{rand} thermal stochastic force



$$\langle F_{rand}(t) \rangle = 0 \qquad \langle F_{rand}(t_1), F_{rand}(t_2) \rangle = \delta(t_1 - t_2)$$

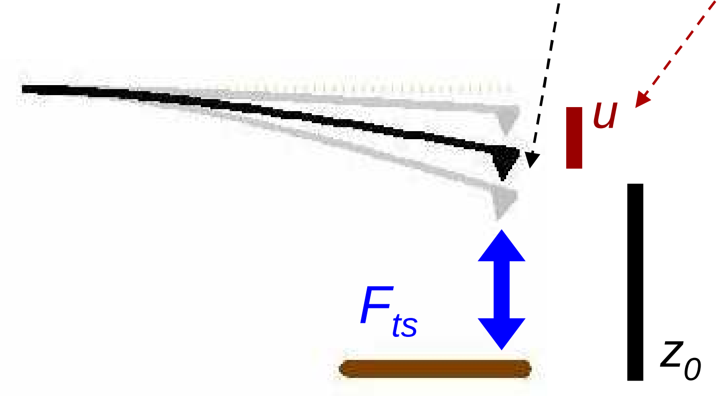
V.L.Mironov *Fundamentals of SPM* (2004)
F.J.Giessibl, *Rev. Mod. Phys.* **75**, 949 (2003)
D.T.Gillespie, *Am. J. Phys.* **61**, 1077 (1993)

Interacting cantilever

Cantilever near the surface → F_{ts} tip-sample interaction force

$$m^* \frac{d^2u}{dt^2} + \gamma \frac{du}{dt} + ku = F_{rand} + F_{ts}(z)$$

$$F_{ts}(z) = F_{ts}(z_0) + \frac{\partial F_{ts}(z_0)}{\partial z} u + \ldots$$



- The constant force $F_{ts}(z_0)$ only displaces the equilibrium position z_0 (static deflection)
- The force derivative $\partial F_{ts} / \partial z$ influences the cantilever oscillations

V.L.Mironov *Fundamentals of SPM* (2004)
F.J.Giessibl, *Rev. Mod. Phys.* **75**, 949 (2003)

Small amplitude oscillations →

force gradient $\partial F_{ts} / \partial z$ constant during the cantilever oscillations

$$m^* \frac{d^2u}{dt^2} + \gamma \frac{du}{dt} + ku = F_{rand} + F_{ts}(z_0) + \frac{\partial F_{ts}(z_0)}{\partial z} u$$

$$m^* \frac{d^2u'}{dt^2} + \gamma \frac{du'}{dt} + k^* u' = F_{rand} \qquad k^* = k - \frac{\partial F_{ts}}{\partial z}$$

u' cantilever displacement from the new equilibrium position z_0

The tip-sample force gradient is directly related to the effective spring constant k^*

V.L.Mironov *Fundamentals of SPM* (2004)
F.J.Giessibl, *Rev. Mod. Phys.* **75**, 949 (2003)

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Tip-sample interaction

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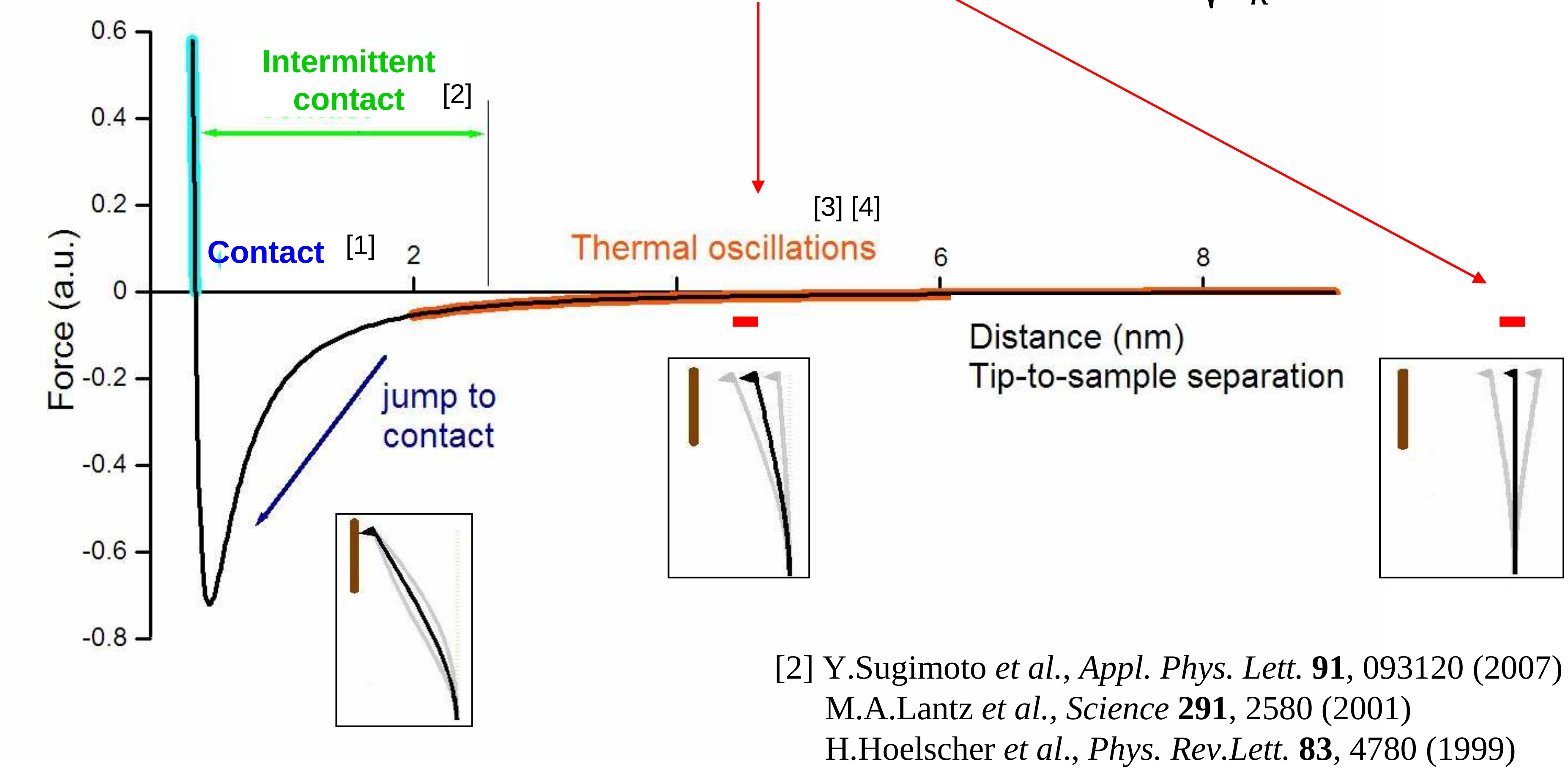
Thermal vibrations amplitude $\sqrt{\langle u^2 \rangle} = \sqrt{\frac{k_B T}{k}}$ < 0.2 nm

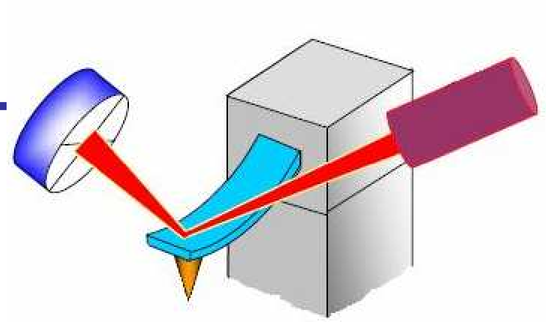


[2] Y.Sugimoto *et al.*, *Appl. Phys. Lett.* **91**, 093120 (2007)
M.A.Lantz *et al.*, *Science* **291**, 2580 (2001)
H.Hoelscher *et al.*, *Phys. Rev.Lett.* **83**, 4780 (1999)
[3] D.O.Koralek *et al.*, *Appl. Phys. Lett.* **76**, 2952 (2000)
A.Roters *et al.*, *J. Phys. Condens. Matter* **8,** 7561 (1996)
[4] J. Buchoux *et al.*, *Nanotechn.* **20,** 475701 (2009)

[1] O.Pfeiffer *et al.*, *Phys. Rev. B* **65**, 161403 (2002)
T.Drobek *et al.*, *Phys. Rev. B* **64**, 045401 (2001)

Experimental setup

Quadrant PhotoDiode

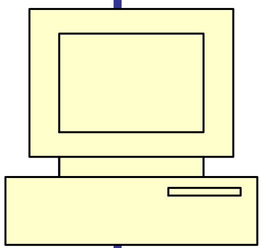


Laser

Cantilever tilt 15° → Senitivity correction [*]

Acquisition system

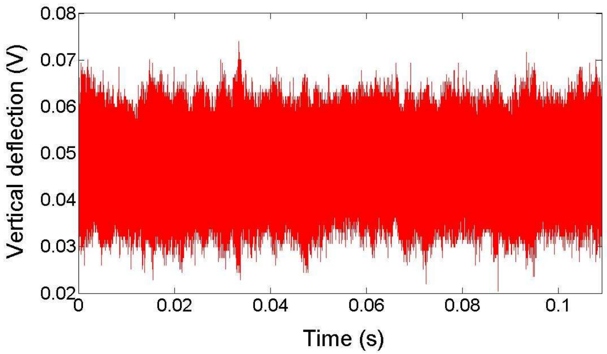
- Bandwidth: > 1 MHz
- Buffer memory: 128MS
- Sensitivity: 50-200 nm/V



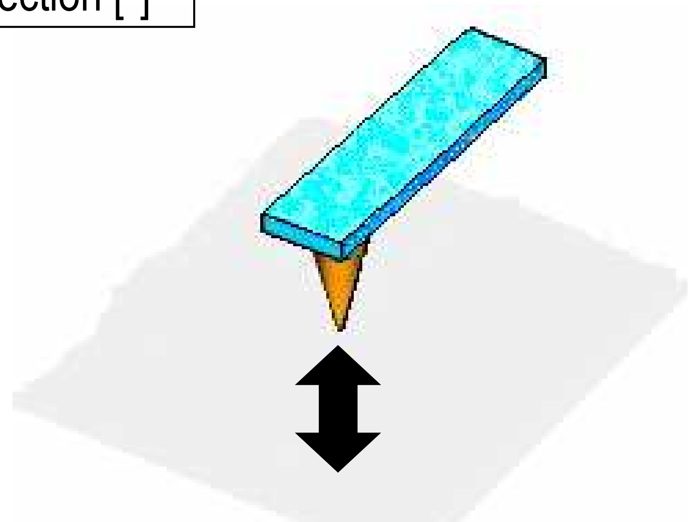
Time signal

Vertical → flexion

Lateral → torsion



Off-line analysys



Tip-sample interaction

[*] J.L.Hutter, *Langmuir* **21**, 2630 (2005)

- One temporal trace of the cantilever thermal motion
- Three different approaches to measure the tip-sample interaction
  - Frequency shift method
  - Potential from Boltzmann distribution
  - Mean-square displacement from power spectral density

- Introduction
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Free cantilever's flexural modes

Boundary conditions

$$f_n^F = \alpha_n^2 \frac{h}{L^2}\sqrt{\frac{E}{12\rho}}$$

Geometrical parameters

Material properties

Mode	1	2	3	4	5
f_n (kHz)	10.8	74.3	215	431	724
f_n / f_1 (exp.)		6.9	20	40	67
f_n / f_1 (th. [*])		6.27	17.55	34.39	56.84

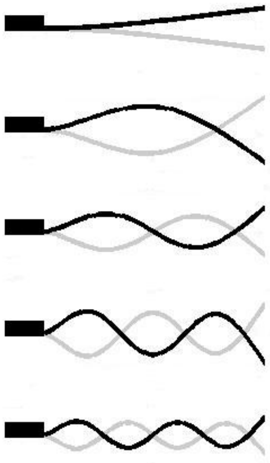
1st

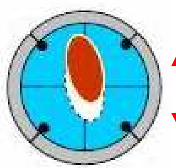
2nd

3rd

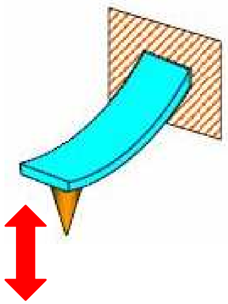
4th

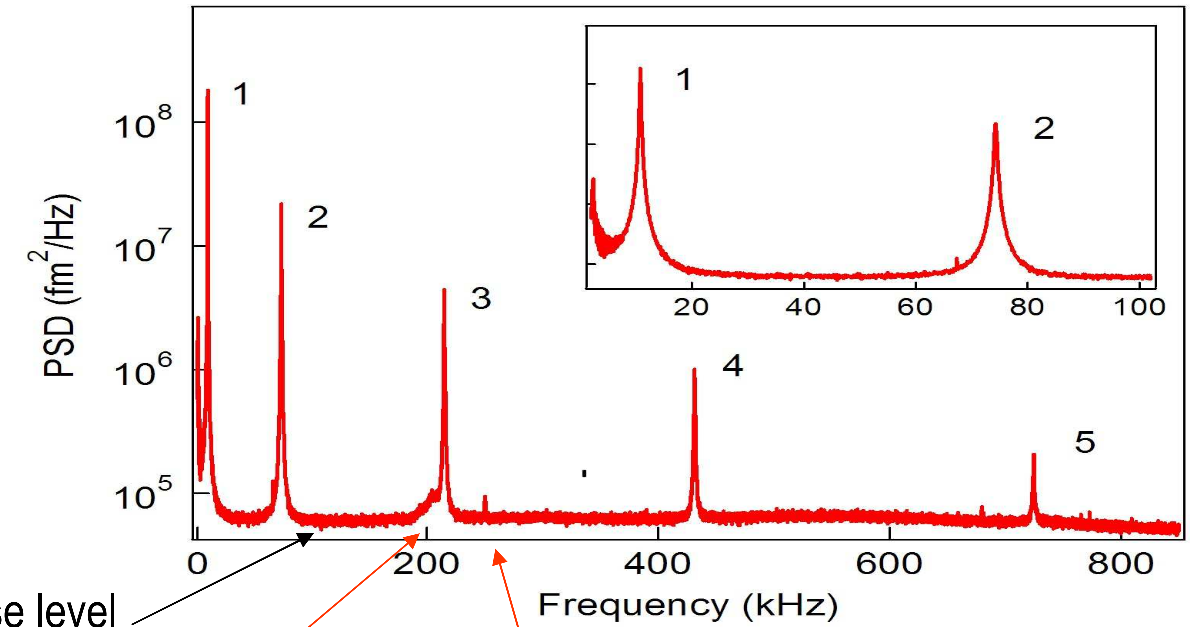
5th





Vertical signal





White noise level
≈ 250 fmHz$^{-0.5}$

1st Lateral

1st Torsional

[*] H.J.Butt *et al.*, *Nanotecnology* **6**, 1 (1995)

Frequency shift method

The resonance frequency of free cantilever is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m^*}}$$

k spring constant, *m** effective mass

The tip-sample interaction F_{ts} changes the resonance frequency.
For small amplitude oscillations

$$k^* = k - \frac{\partial F_{ts}}{\partial z} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k^*}{m^*}}$$

k* effective spring constant

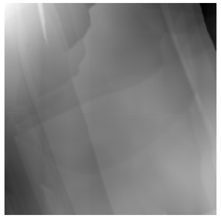
The frequency shift is directly related to the tip-sample force gradient

$$f = f_0 + \Delta f$$

$$\boxed{\frac{\partial F_{ts}}{\partial z} = -2k\frac{\Delta f}{f_0}}$$

Y.Martin *et al.*, *J. Appl. Phys.* **61**, 4723 (1987)
F.J.Giessibl, *Rev. Mod. Phys.* **75**, 949 (2003)

Thermal PSD approaching the HOPG surface



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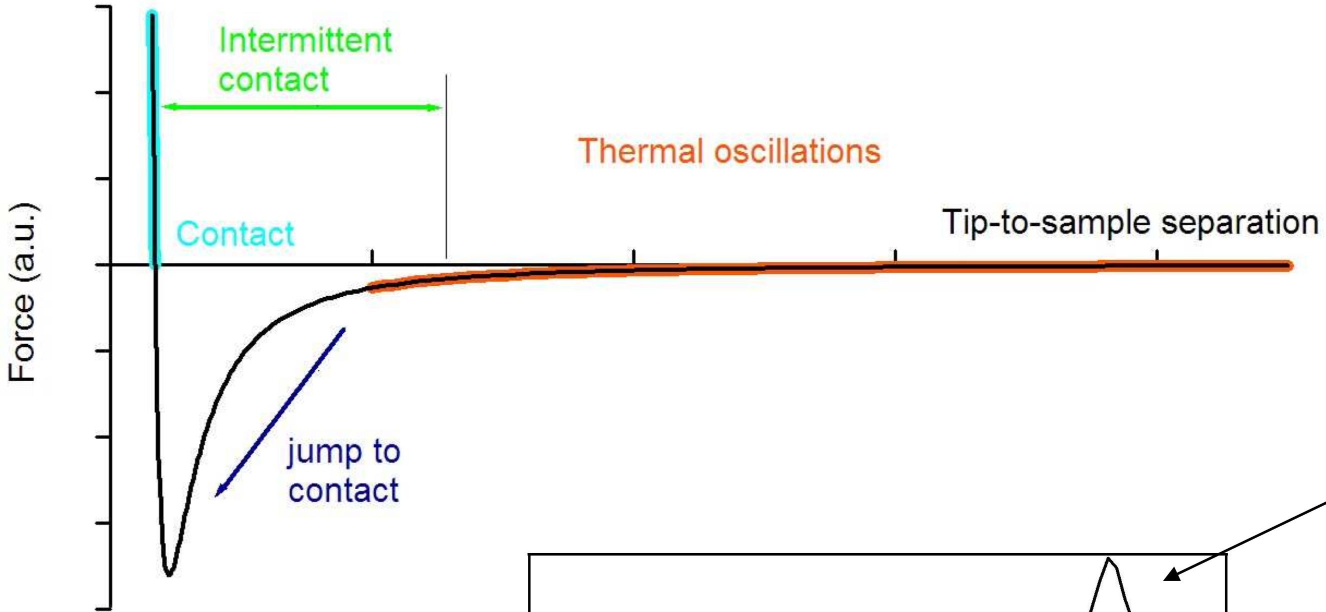
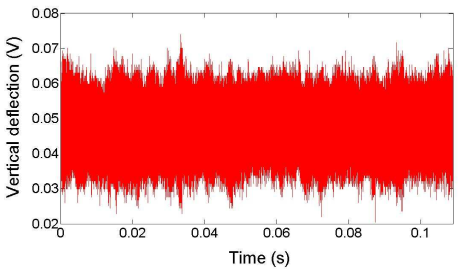
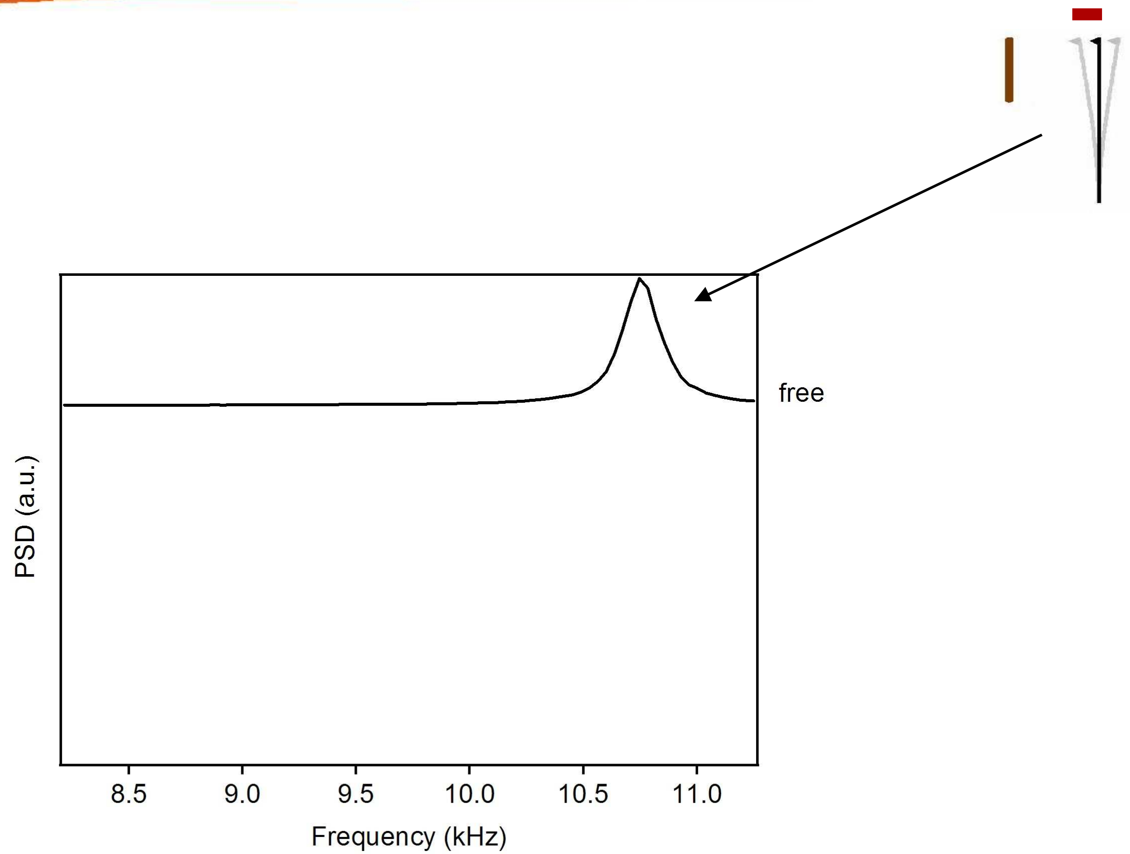




Power Spectral Density



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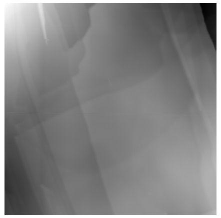

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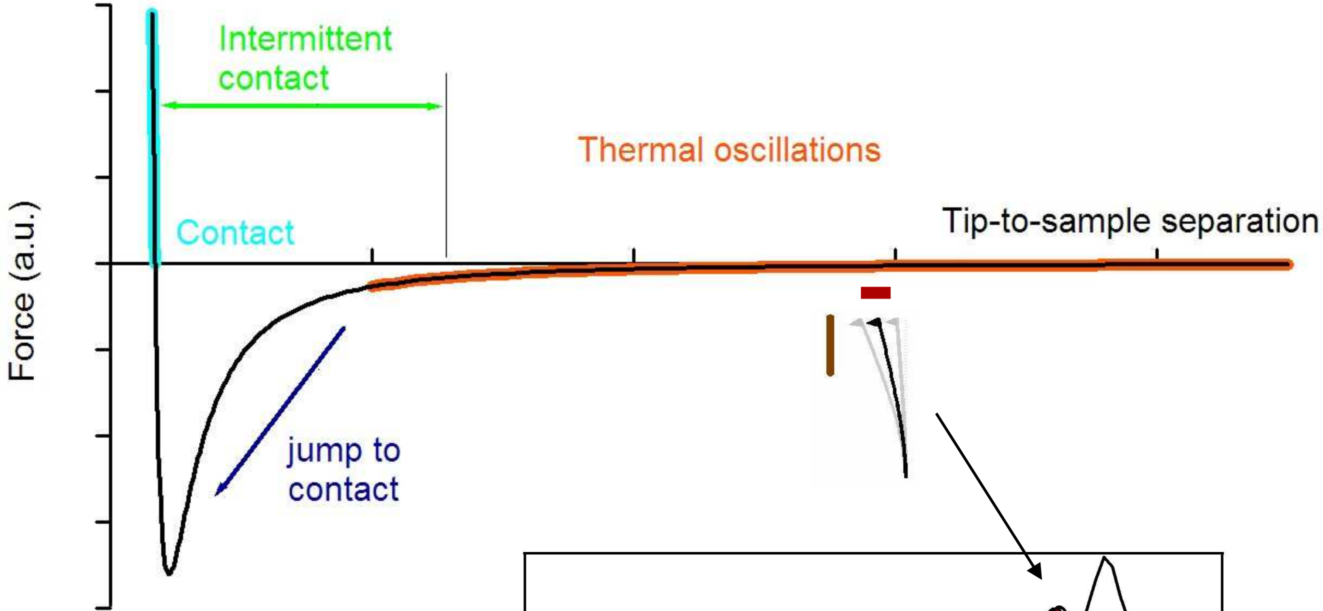

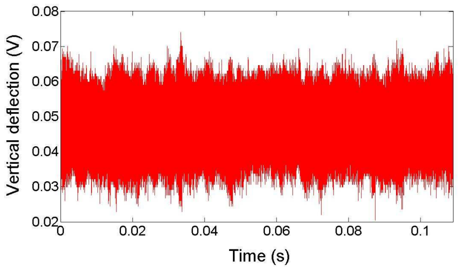

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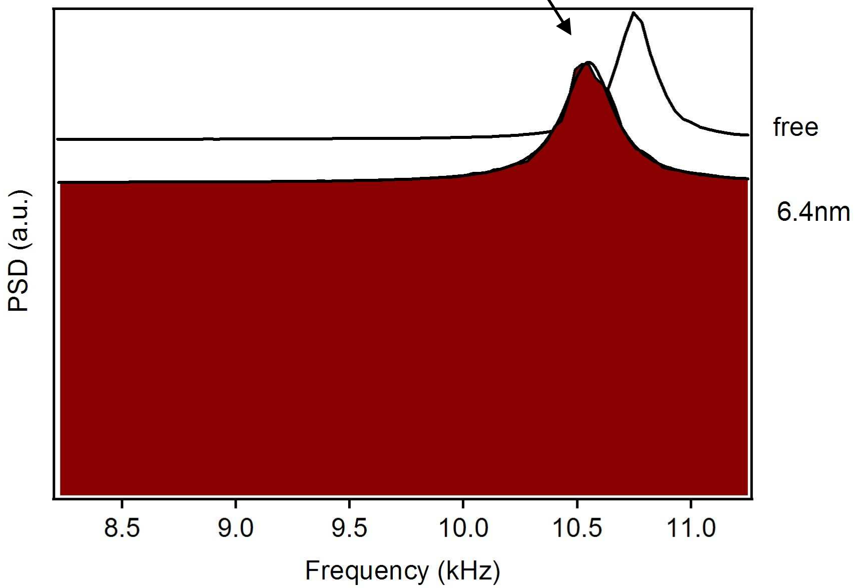

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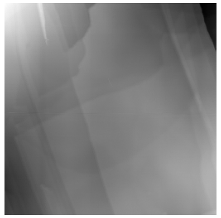

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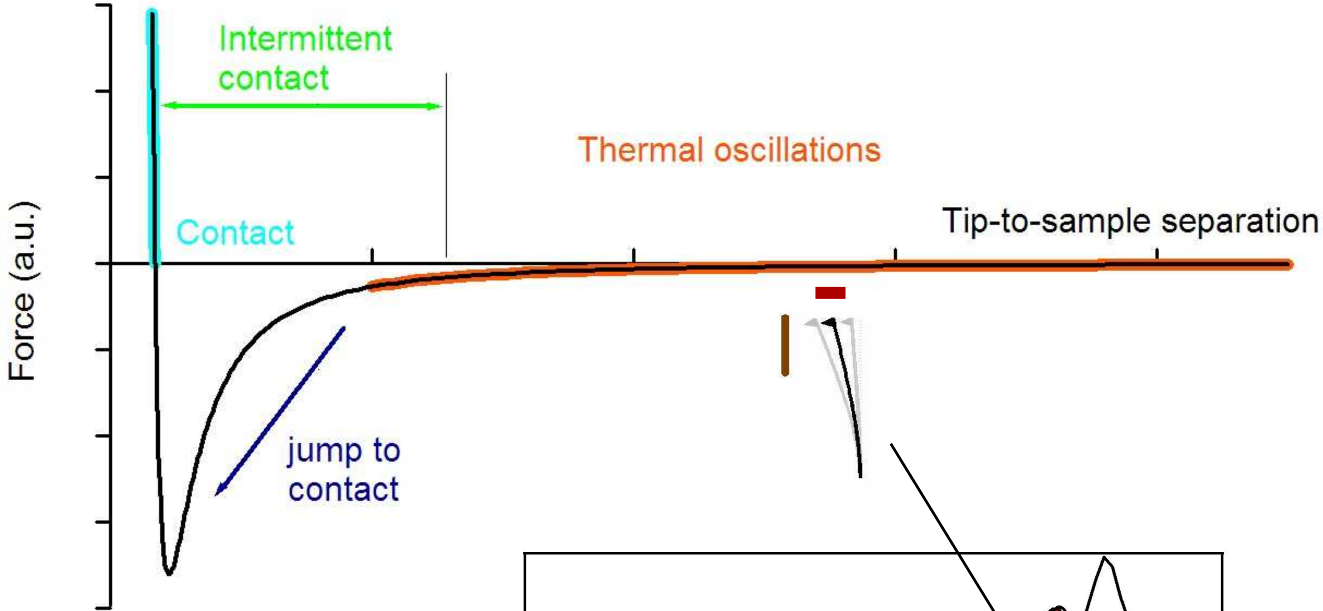

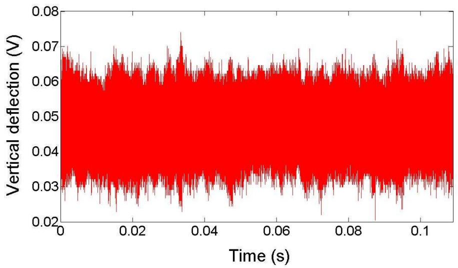

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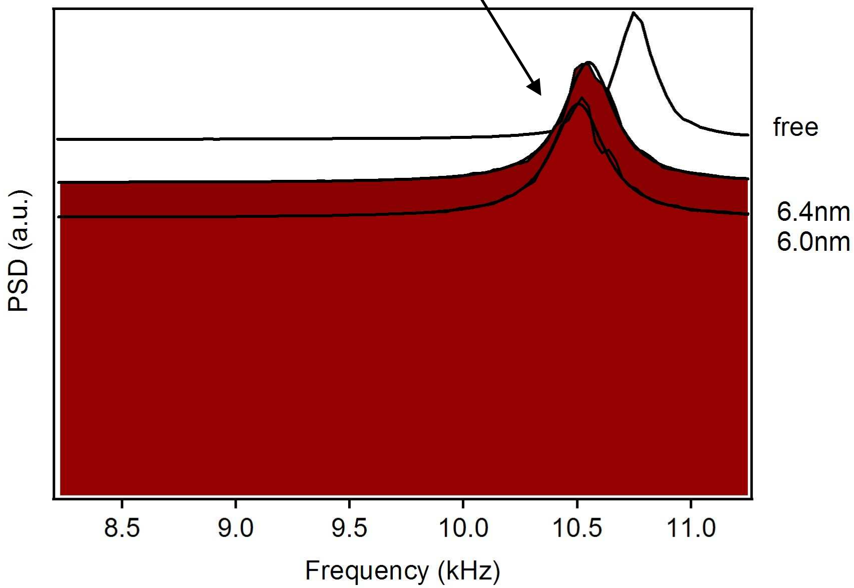
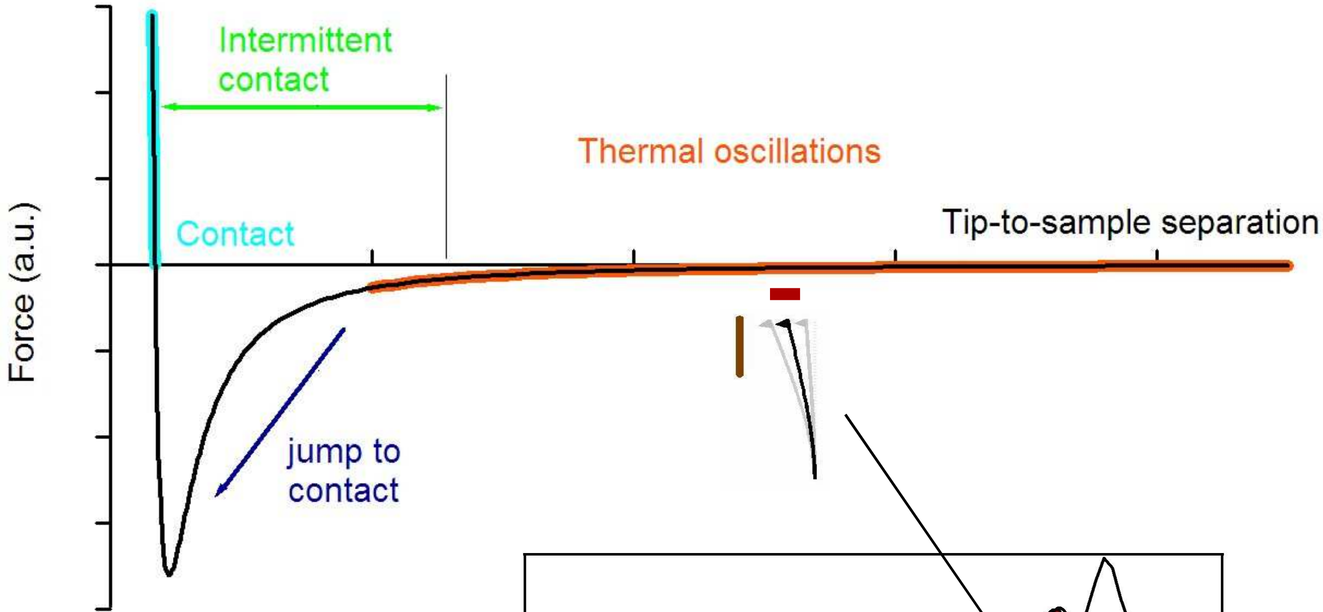
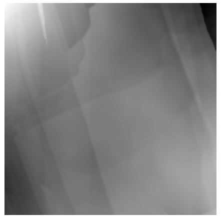


Thermal PSD approaching the HOPG surface





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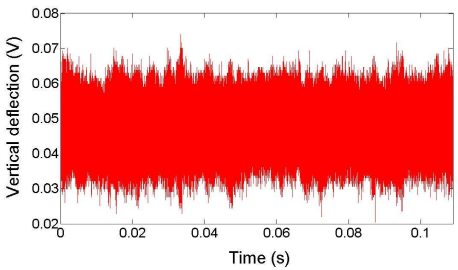

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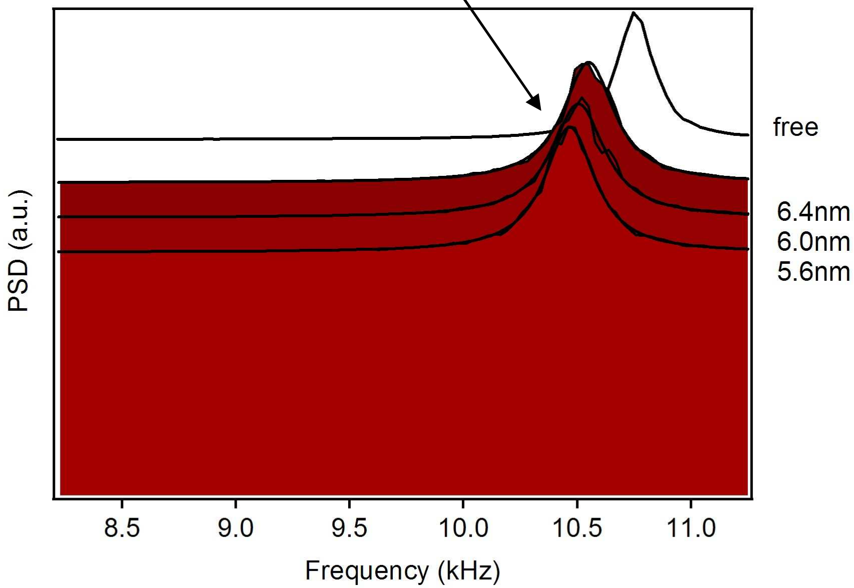

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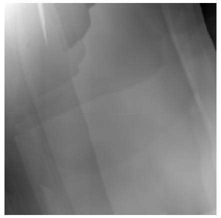

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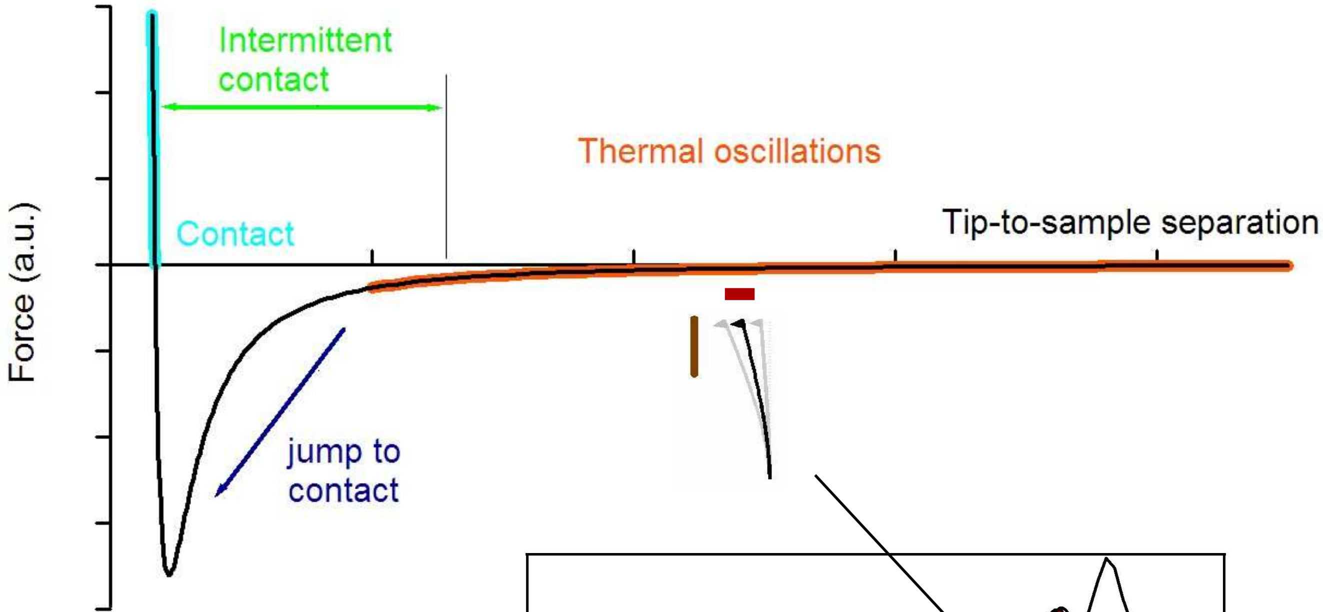

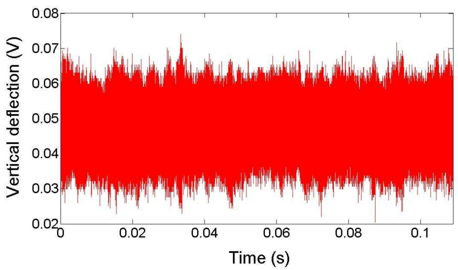

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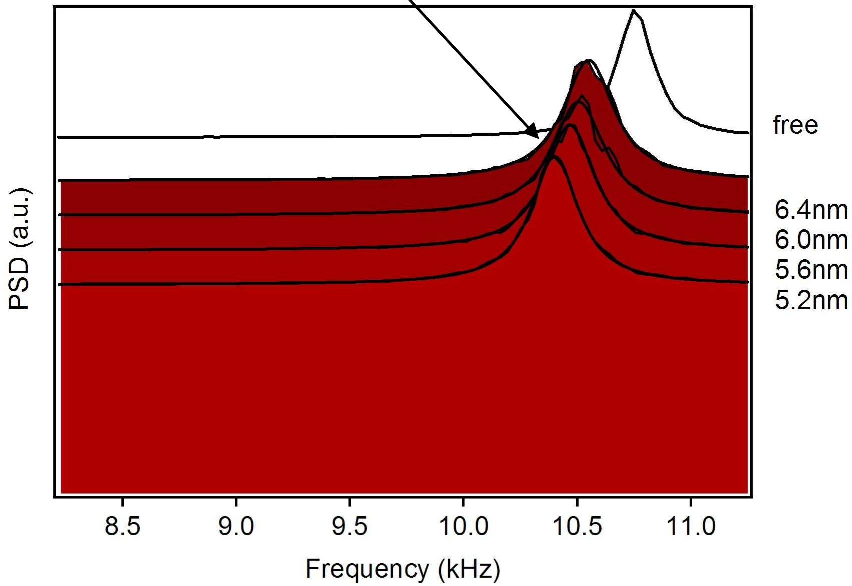

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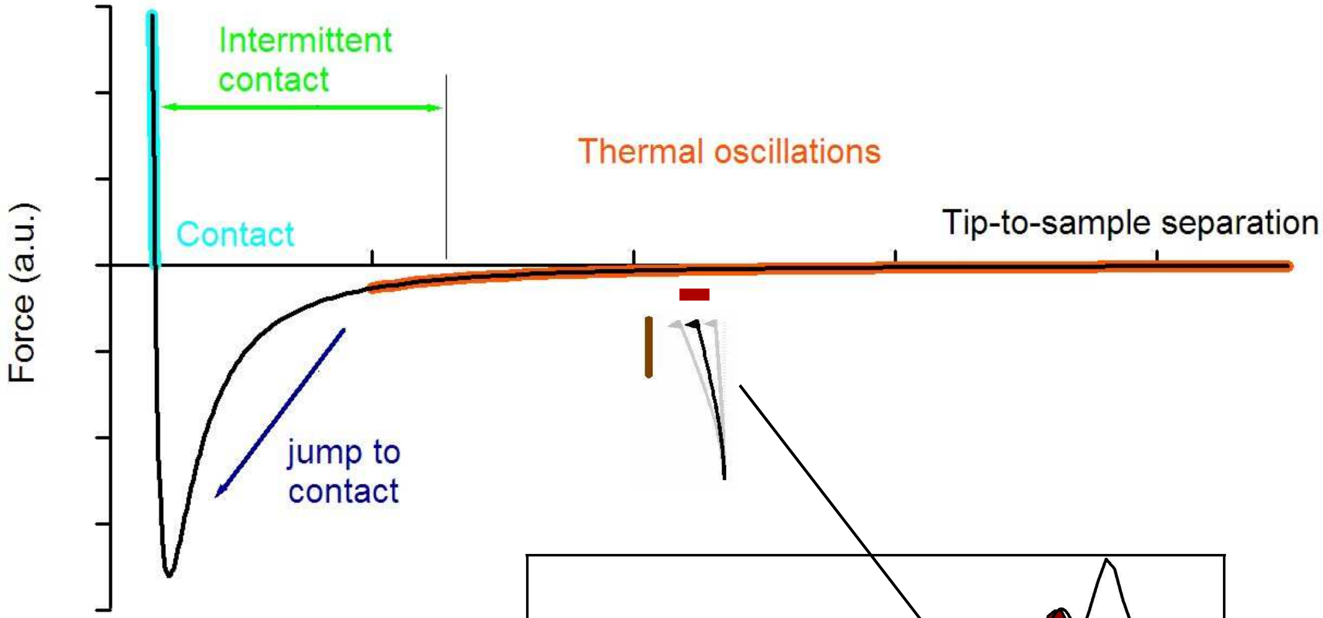

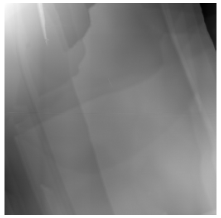

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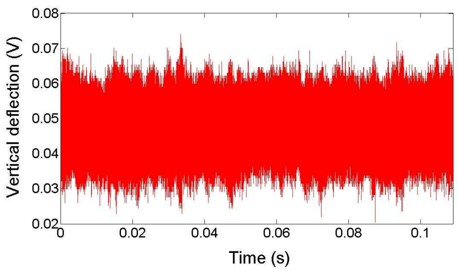

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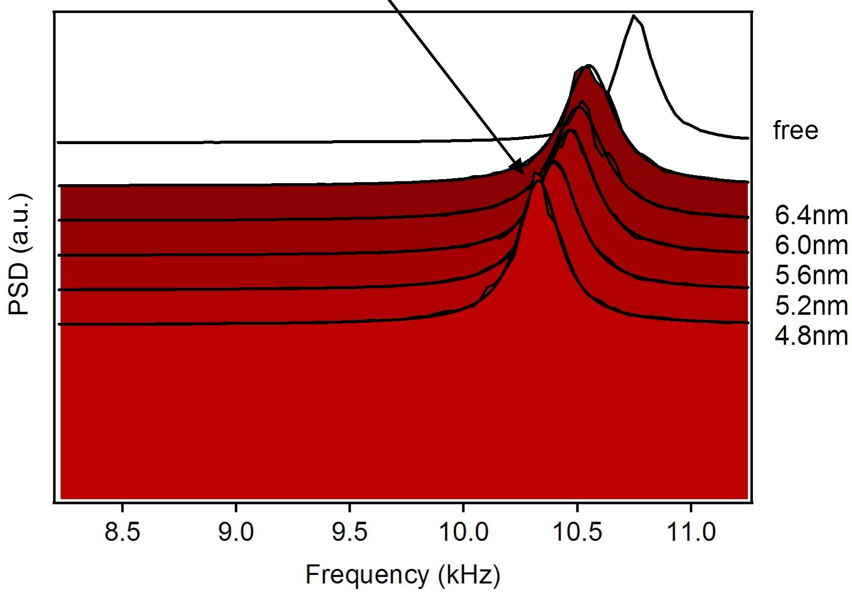

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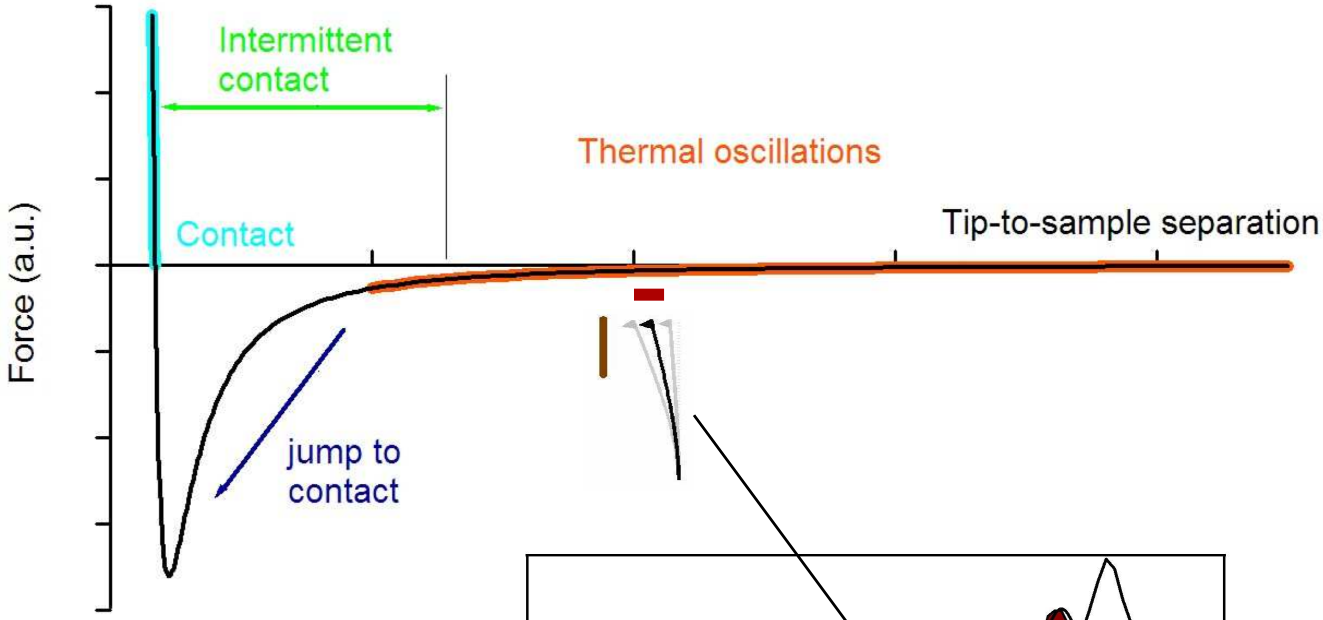

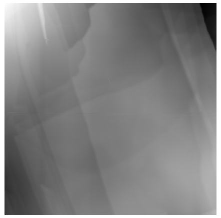

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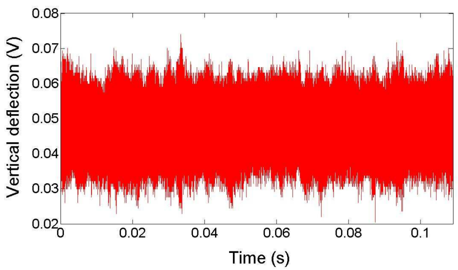

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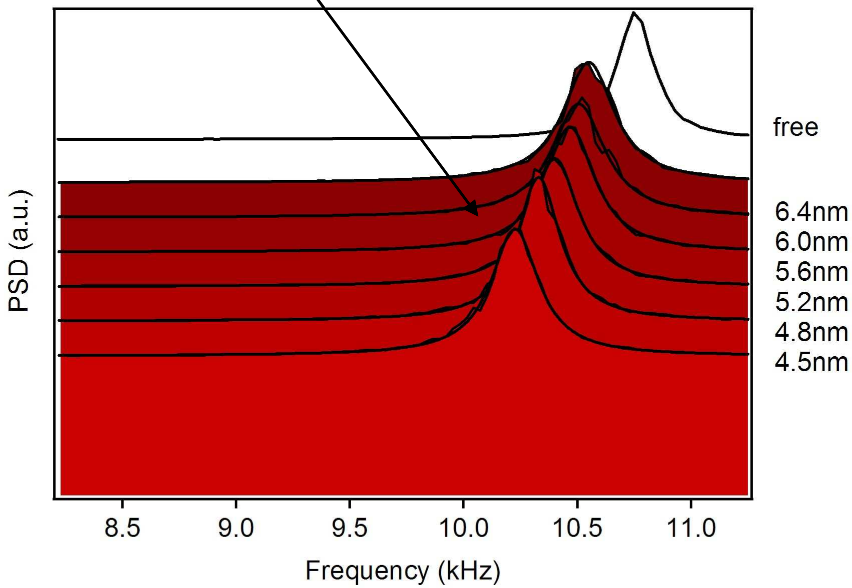

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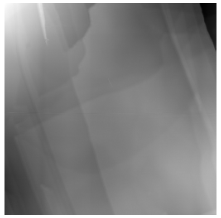

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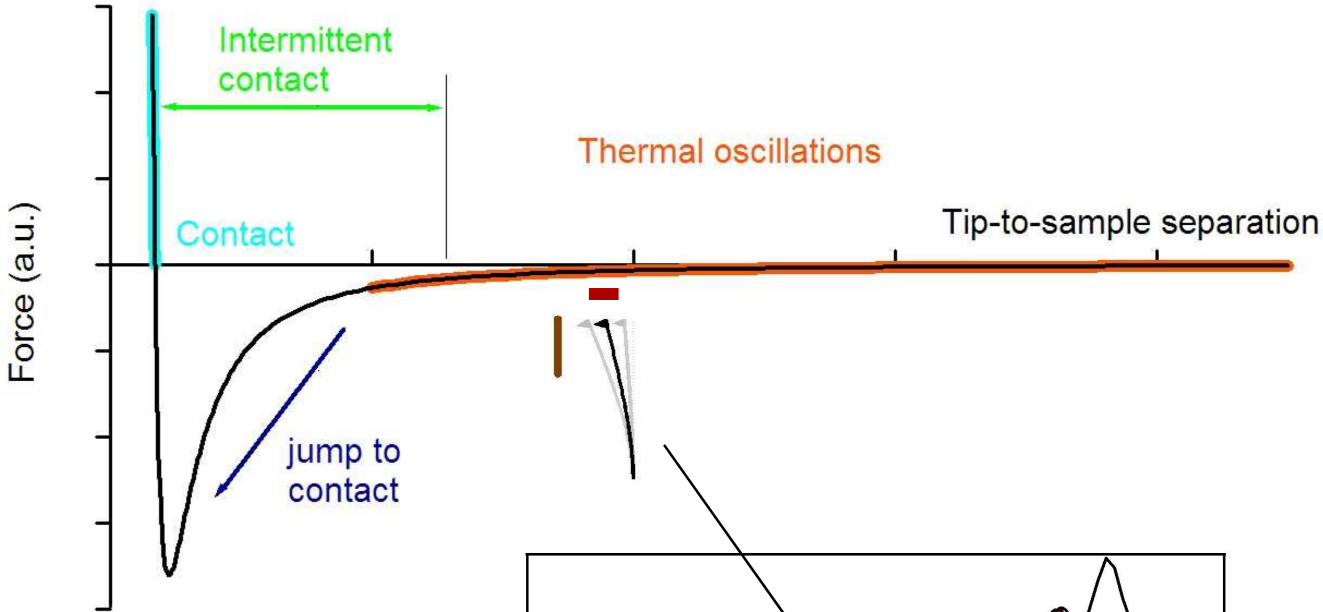

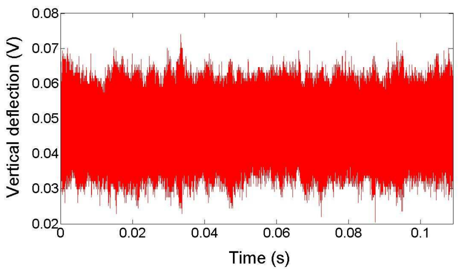

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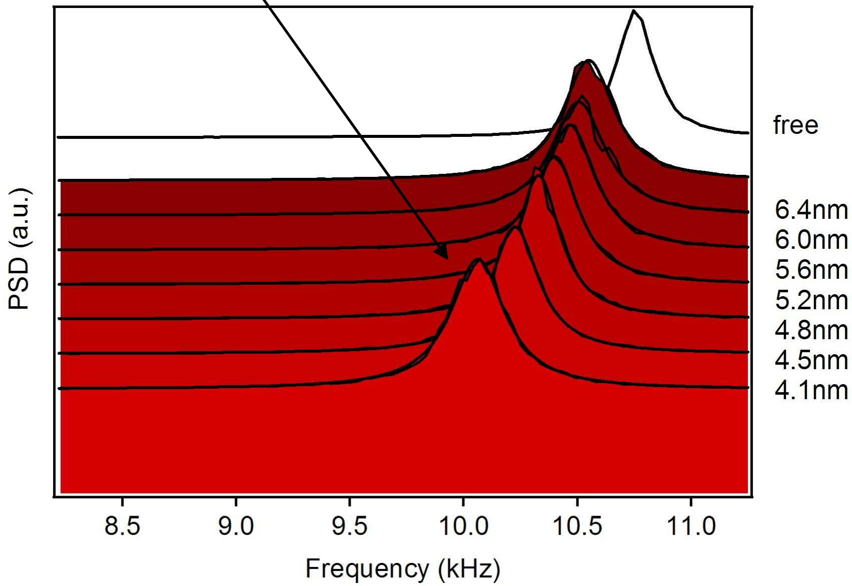

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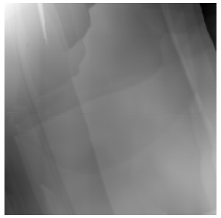

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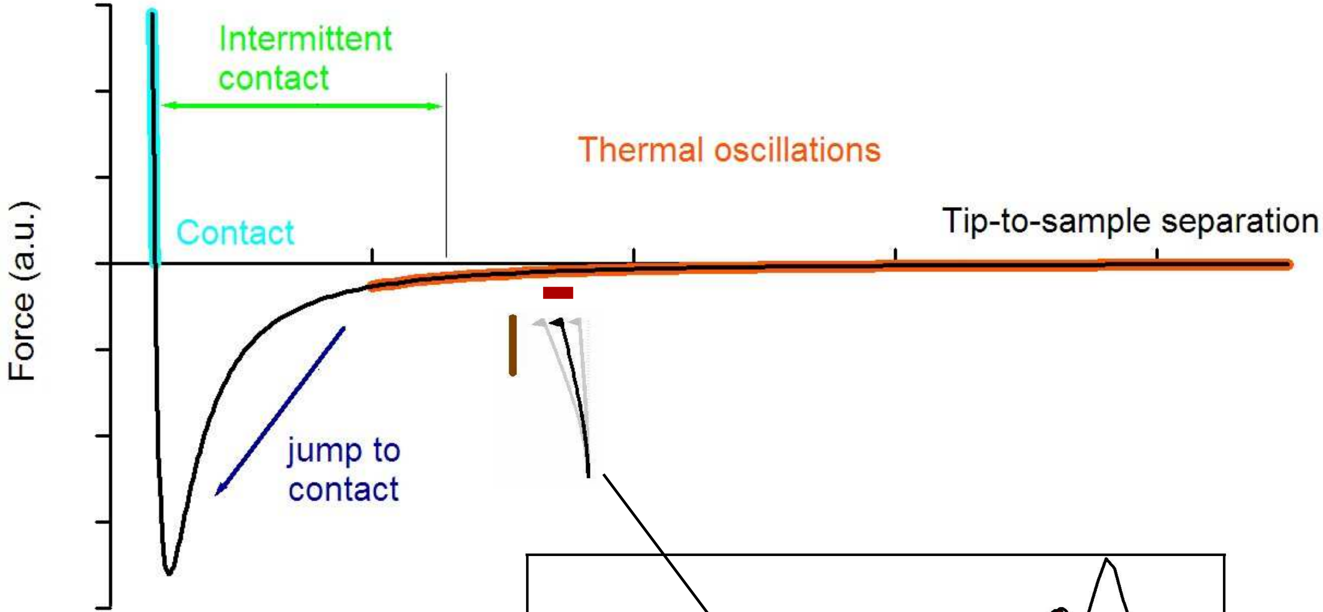

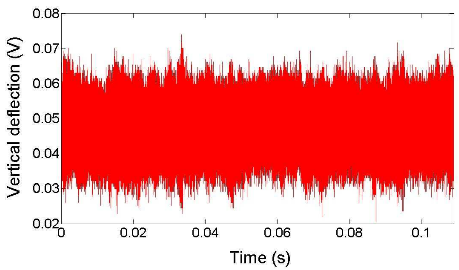

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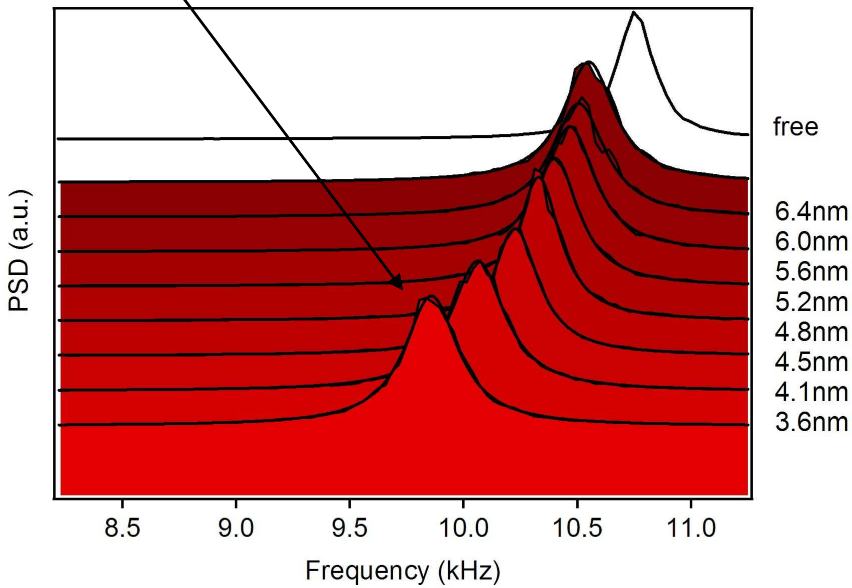

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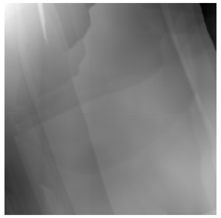

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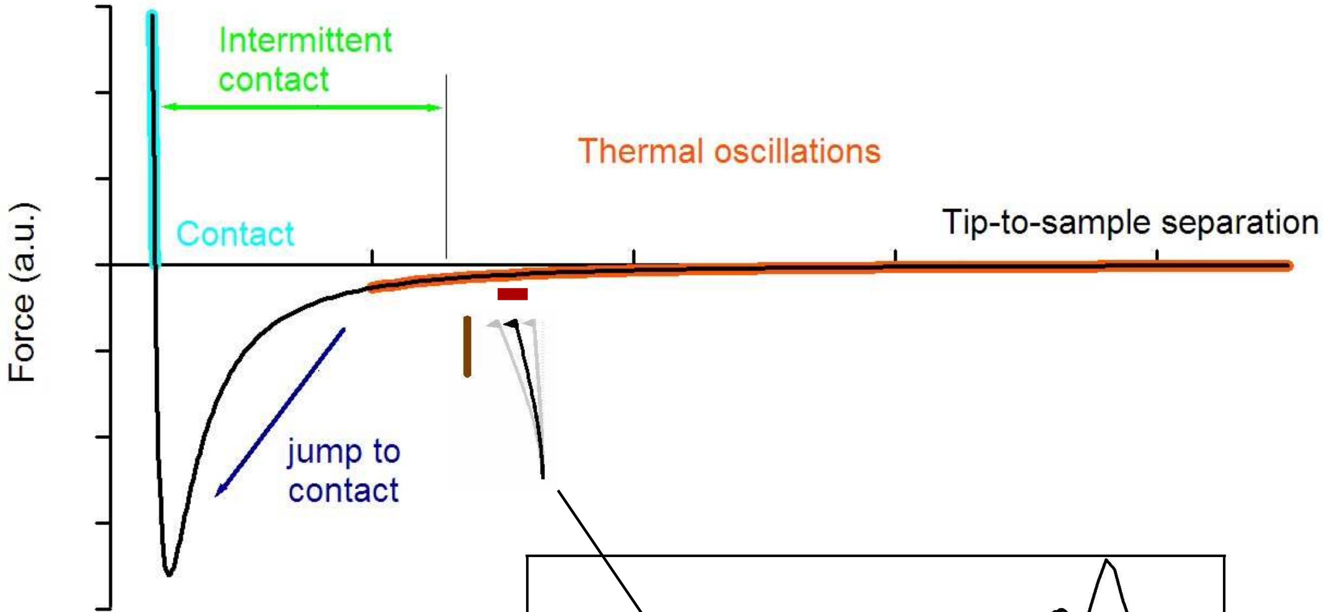
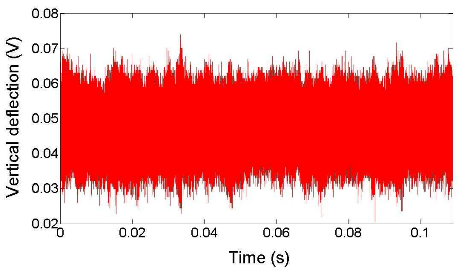
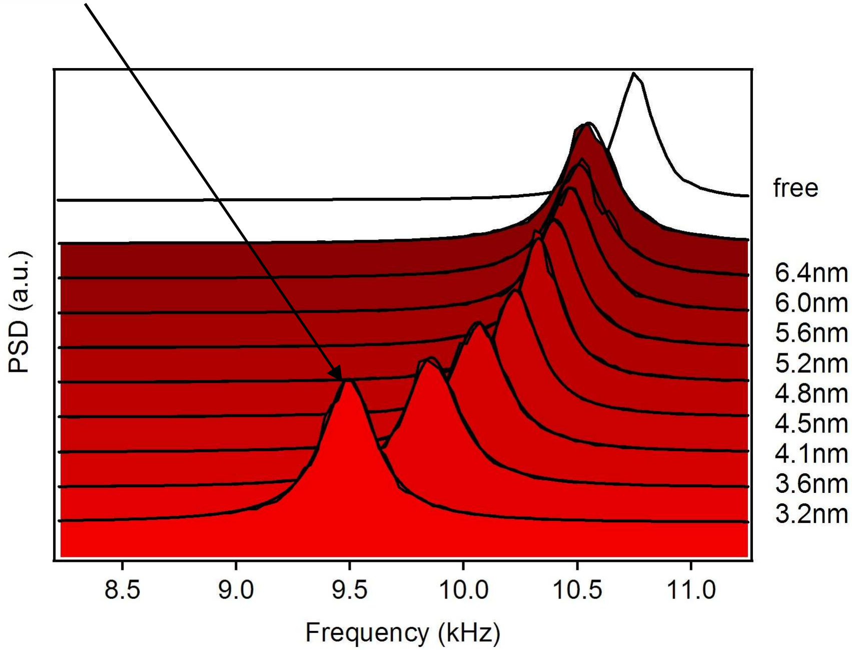
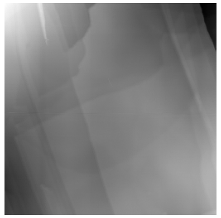




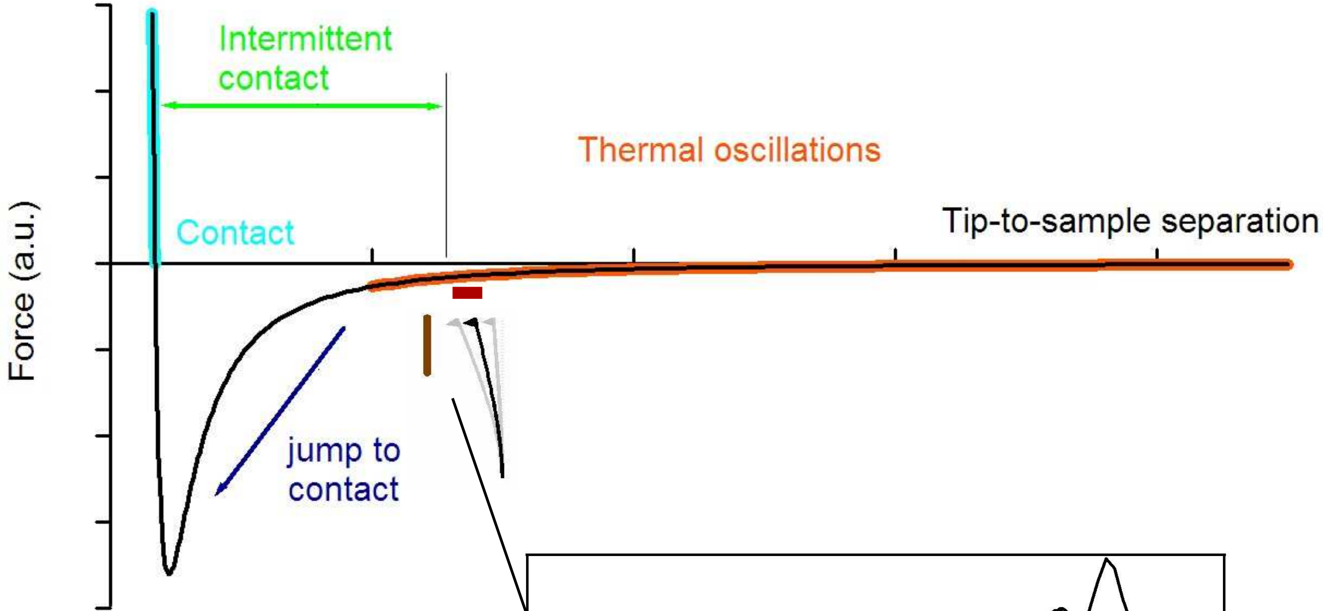
Power Spectral Density

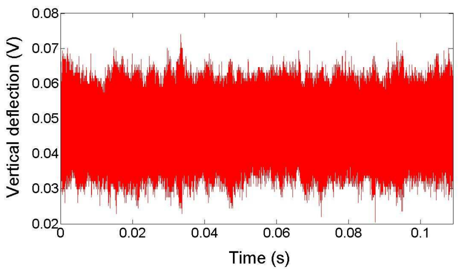


Thermal PSD approaching the HOPG surface

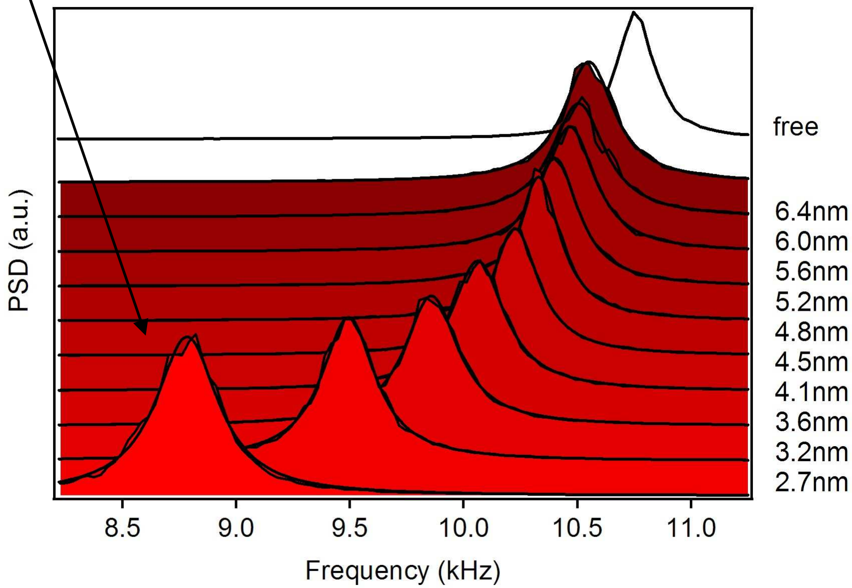


HOPG

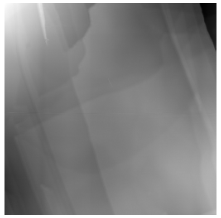




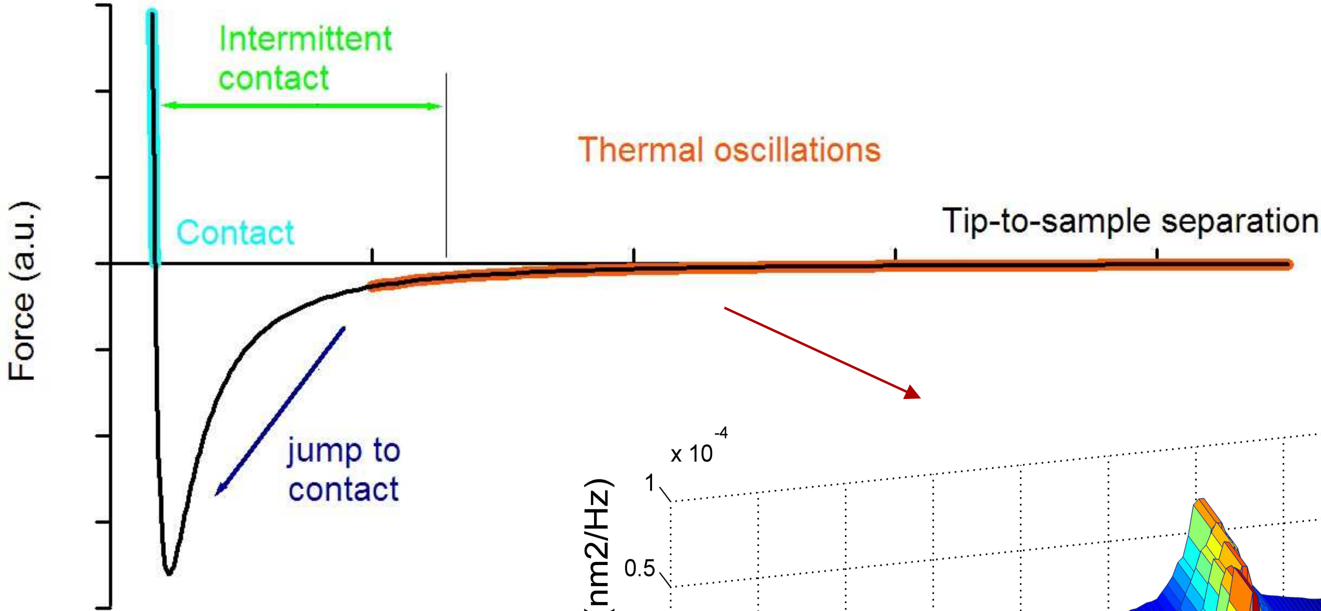
Power Spectral Density

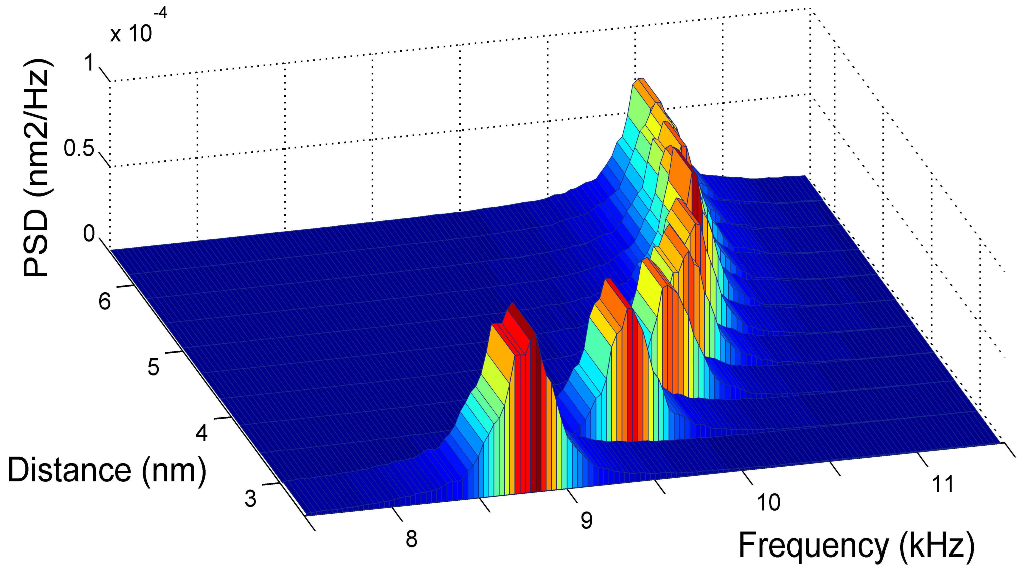


Thermal PSD approaching the HOPG surface



HOPG





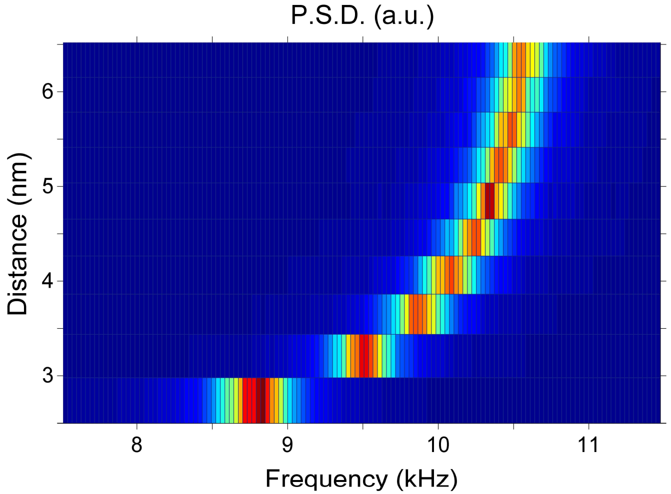
The jump-to-contact occurs when

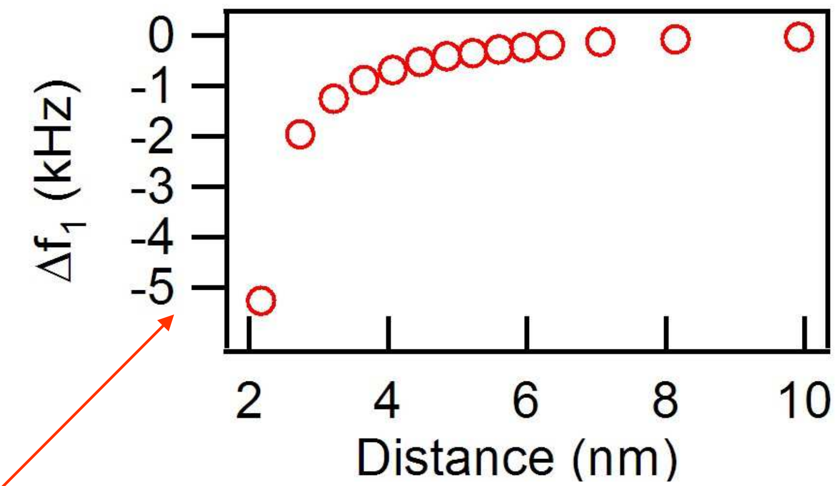
$$\left.\frac{\partial^2 V_{ts}}{\partial z^2}\right|_{max} = -\left.\frac{\partial F_{ts}}{\partial z}\right|_{max} > k$$

Only long-range forces measurements (Hamaker constant) are possible

F.J.Giessibl, *Rev. Mod. Phys.* **75**, 949 (2003)

First flexural mode frequency shift

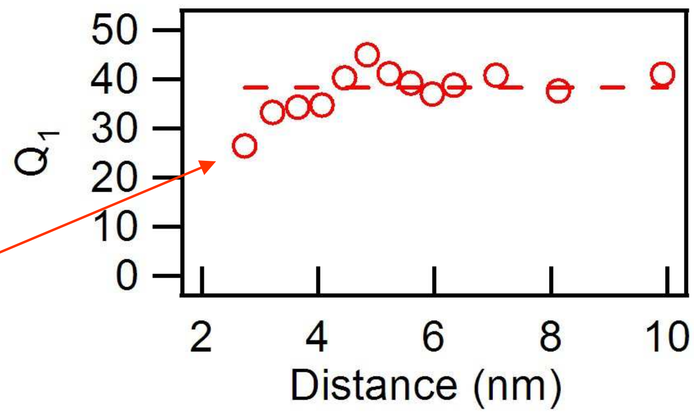




Resonance frequency decreases →
Attractive tip-sample force

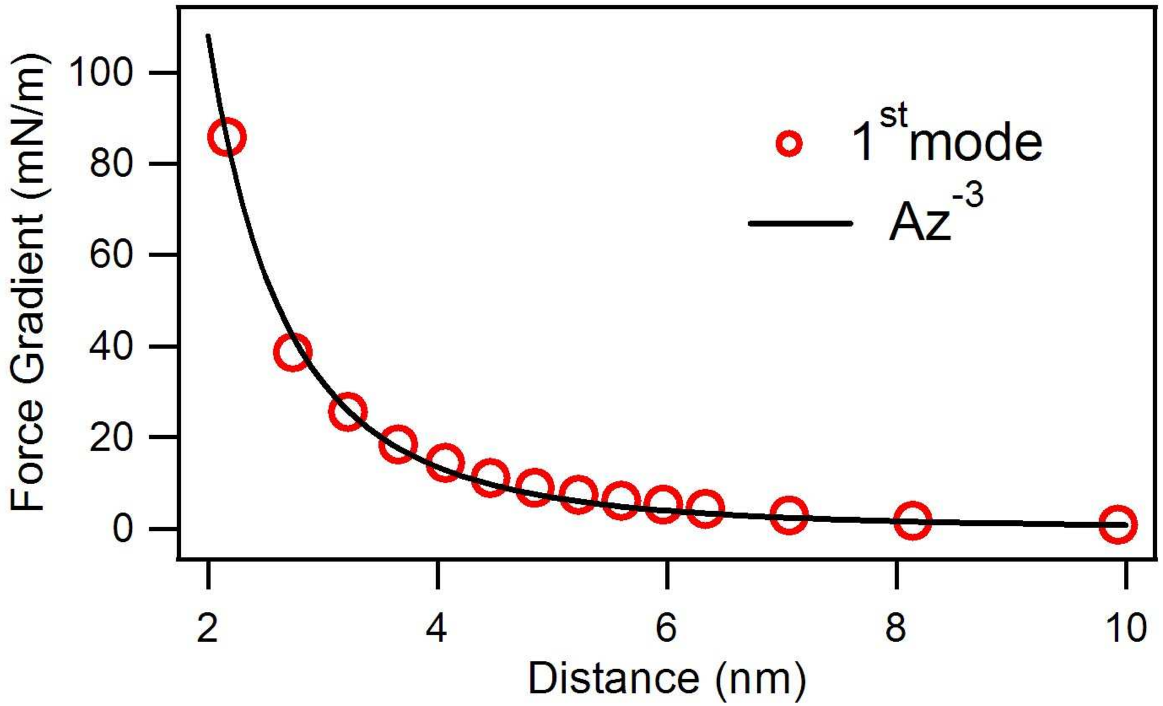
Q-factor constant →
Non-dissipative interaction

- Tip-sample not in contact
- Dissipation mechanism probably due to some interaction of the very end of the tip with the surface



Non retarded van der Waals forces

$$\frac{\partial F_{ts}}{\partial z} = -2k\,\frac{\Delta f}{f_0}$$



Non retarded van der Waals force

$$\frac{\partial F_{ts}}{\partial z} = \frac{HR}{3(z - z_0)^3}$$

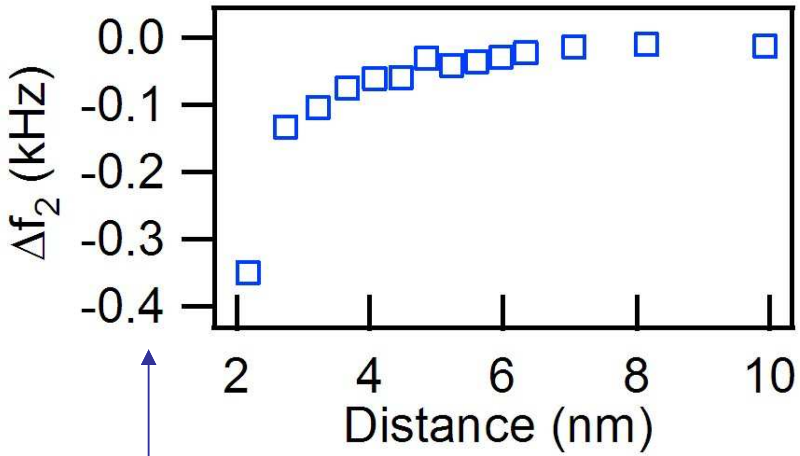
HR = 3 ·10^{-27}J m

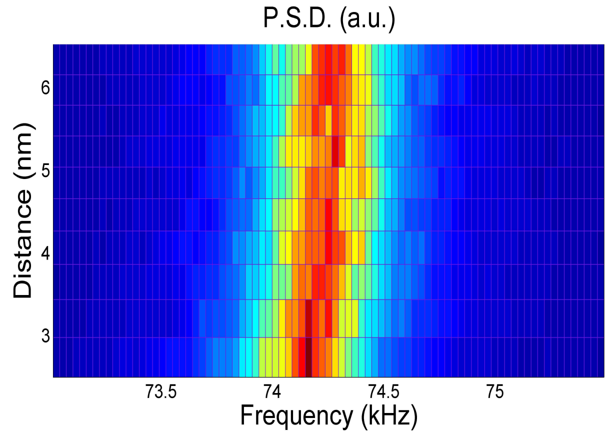
z_0 surface position

H Hamaker constant, R tip radius (≈30 nm), $z - z_0$ tip-sample distance (piezo-tube position, z_0 surface position and cantilever static deflection $\Delta z = F_{ts} / k$)

Butt *et al.*, *Surf. Sci. Rep.* **59**, 1 (2005)

Second flexural mode

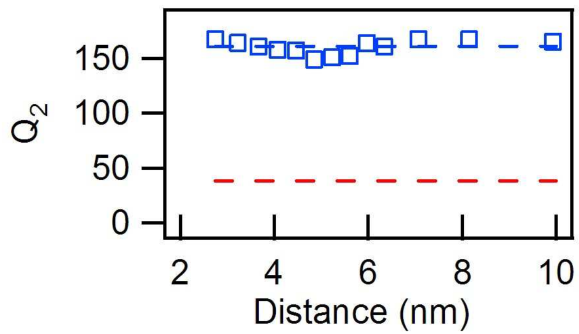




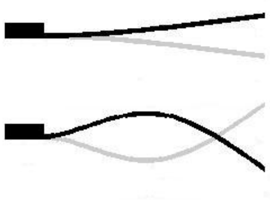
Second mode's stiffness higher than first mode → lower resonance frequency decreases

Flexural mode	1	2	3
***k**$_n$* **(N/m)** [1]	0.12	5.0	40
k_n / k_1 (exp.)		43	350
k_n / k_1 (th. [2])		39	300

Non-dissipative interaction

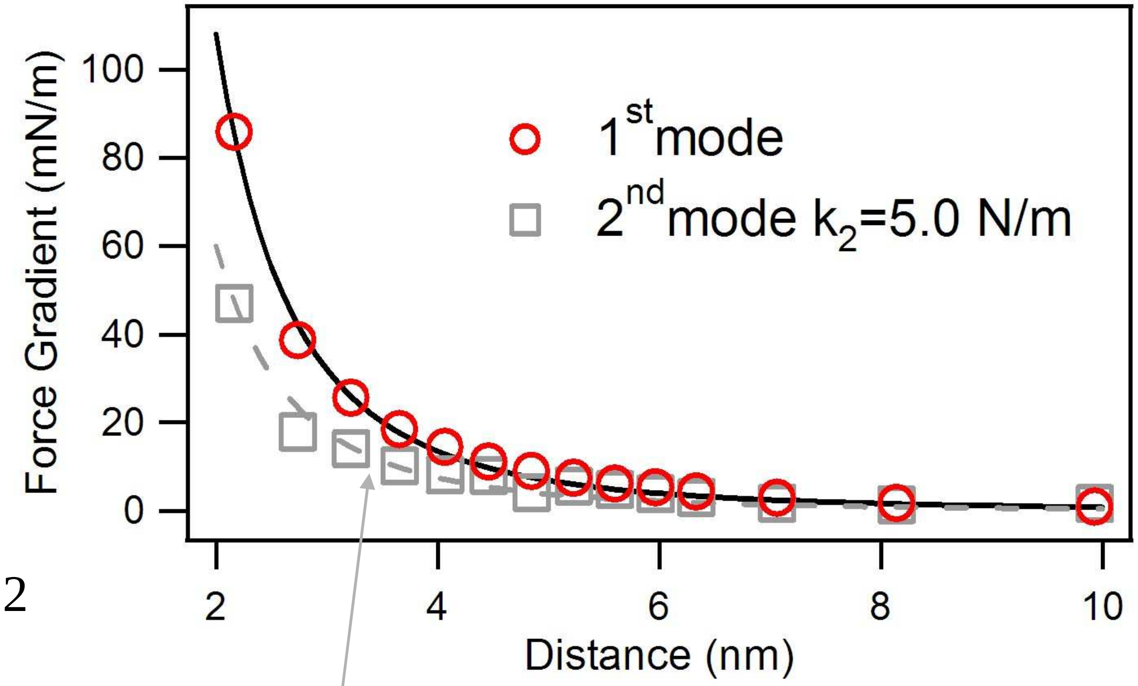


[1] J.E.Sader *et al.*, *Rev. Sci. Ins.* **70**, 3967 (1999)
[2] J.Melcher *et al.*, *Appl. Phys. Lett.* **91**, 053101 (2007)



$$\frac{\partial F_{ts}}{\partial z} = -2k_i \frac{\Delta f_i}{f_{i0}}$$

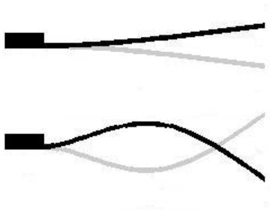
$$i = 1,2$$



Second mode's spring constant k_2

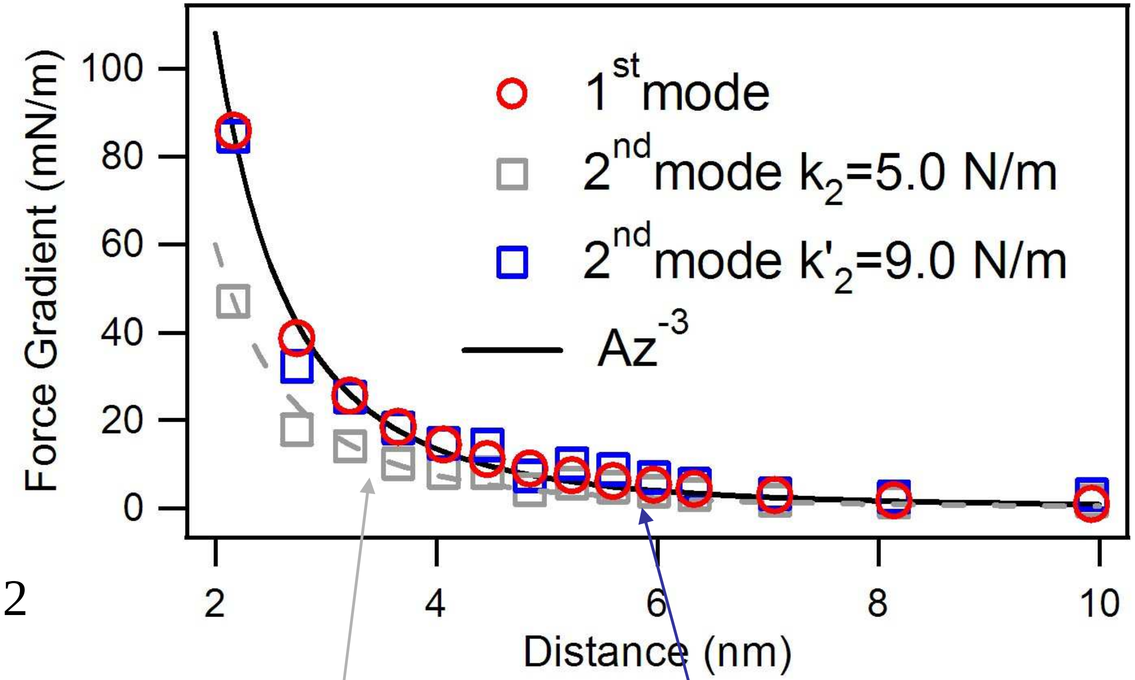
- by Sader method → No matching

First and second flexural modes



$$\frac{\partial F_{ts}}{\partial z} = -2k_i \frac{\Delta f_i}{f_{i0}}$$

$$i = 1,2$$

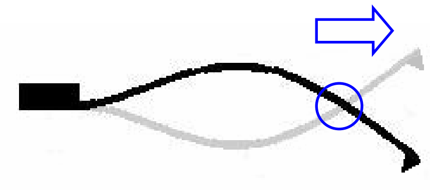


Second mode's spring constant k_2

- by Sader method → No matching
- Stiffer spring constant (tip mass effect) → Good matching

Second mode equivalent stiffness

Tip mass loading



1st: no significant effect (1%)

2nd: the node shifts toward the free end
the cantilever effective length reduces
the equivalent stiffness increases

- A tip mass that is 10% of the cantilever mass nearly doubles the second mode equivalent stiffness. [1]
- Sader method [2] is accurate only for the lower modes.

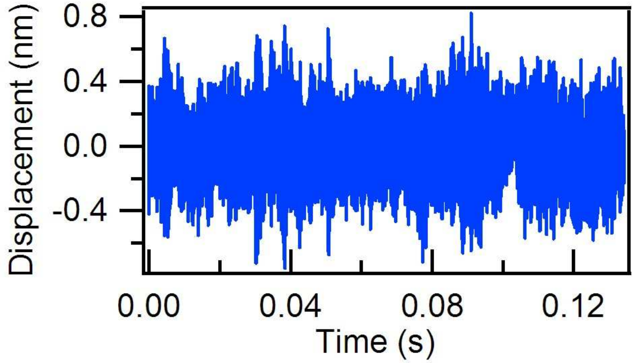
k =0.12 N/m [2]	k_2 Sader meth.	k'_2 fit to k_{ts}	Th. no tip [1]	Th. tip 10% CL [1]
k_2 (N/m)	5.0	**9.0**		
k_2/k	42	**75**	40.2	**74.9**

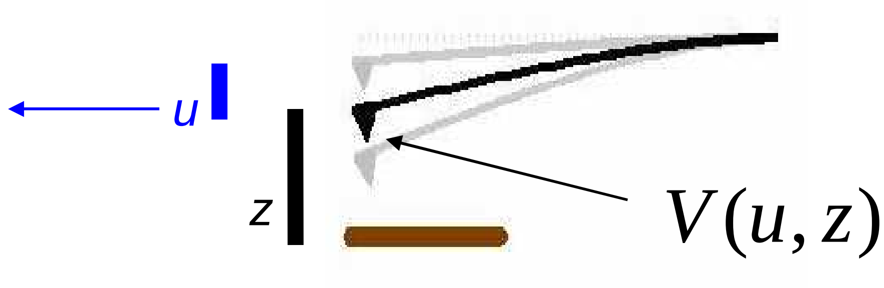
[1] J.Melcher *et al.*, *Appl. Phys. Lett.* **91**, 053101 (2007)
Y.Sugimoto *et al.*, *Appl. Phys. Lett.* **91**, 093120 (2007)

[2] J.E.Sader *et al.*, *Rev. Sci. Ins.* **70**, 3967 (1999)

- Introduction
- Free cantilever thermal vibrations
- Probing the tip-sample interaction
  - Frequency shift
  - Potential from Boltzmann distribution
  - Mean-square displacement from power spectral density

Boltzmann probability distribution



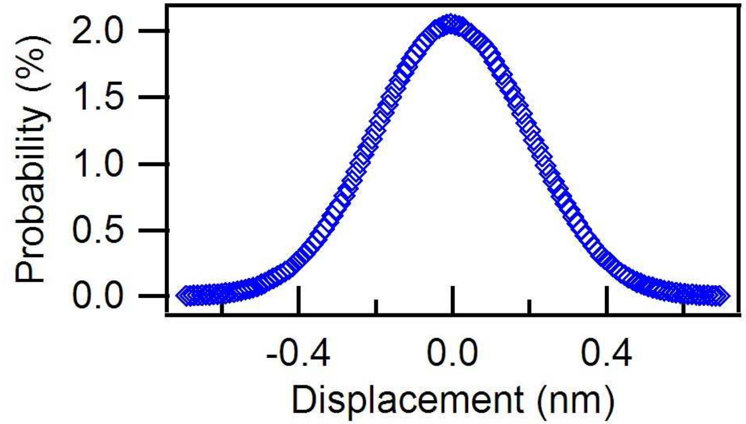


- *p(u,z)* probability of observing the tip at a deflection *u* from the equilibrium position/tip-surface distance *z* →
- number of count at the deflection *u* divided by the total number of count

Boltzmann probability distribution

$$p(u,z) = p_0 e^{-\frac{V(u,z)}{K_B T}}$$

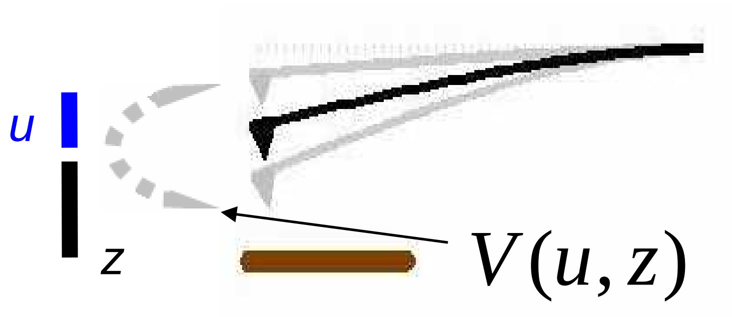
V(u,z) position dependent total potential



W.F. Heinz *et al.*, *J. Phys. Chem. B* **104**, 622 (2000)
D.O.Koralek *et al.*, *Appl. Phys. Lett.* **76**, 2952 (2000)

Harmonic potential

$$V(u,z) = -K_B T \ln \frac{p(u,z)}{p_0}$$



$$V(u,z) = V_C(u,z) + V_{ts}(u,z)$$

V_c cantilever potential

V_{ts} tip-sample interaction potential

$$V = \frac{1}{2} k u^2 - \frac{1}{2} \frac{\partial F_{ts}}{\partial z} u^2 = \frac{1}{2} k^* u^2 \qquad k^* = k - \frac{\partial F_{ts}}{\partial z}$$

Small oscillation amplitude *u* ($\partial F_{ts} / \partial z$ constant) → harmonic potential

Tip-sample force gradient

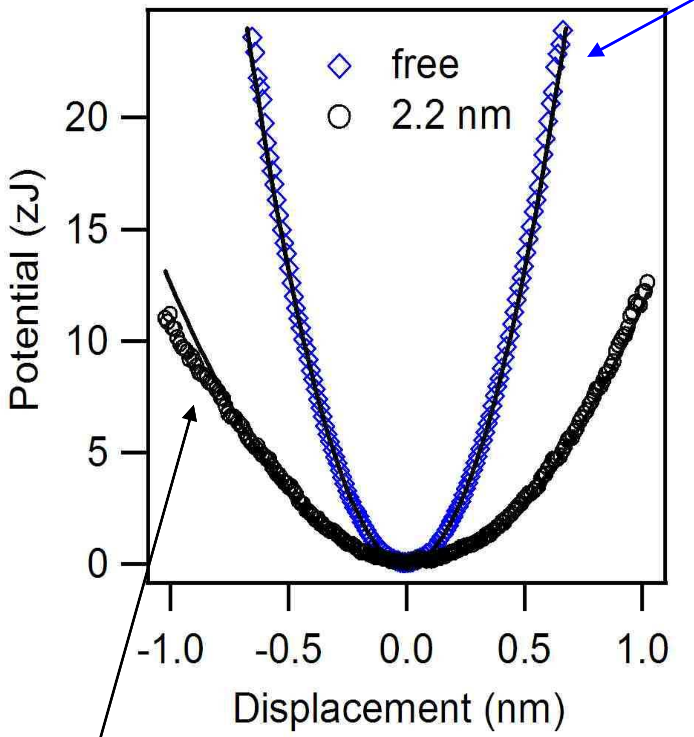
$$\boxed{\frac{\partial F_{ts}}{\partial z} = -\frac{\partial^2 V_{ts}}{\partial s^2}}$$

W.F. Heinz *et al.*, *J. Phys. Chem. B* **104**, 622 (2000)
D.O.Koralek *et al.*, *Appl. Phys. Lett.* **76**, 2952 (2000)

Total and tip-sample potentials

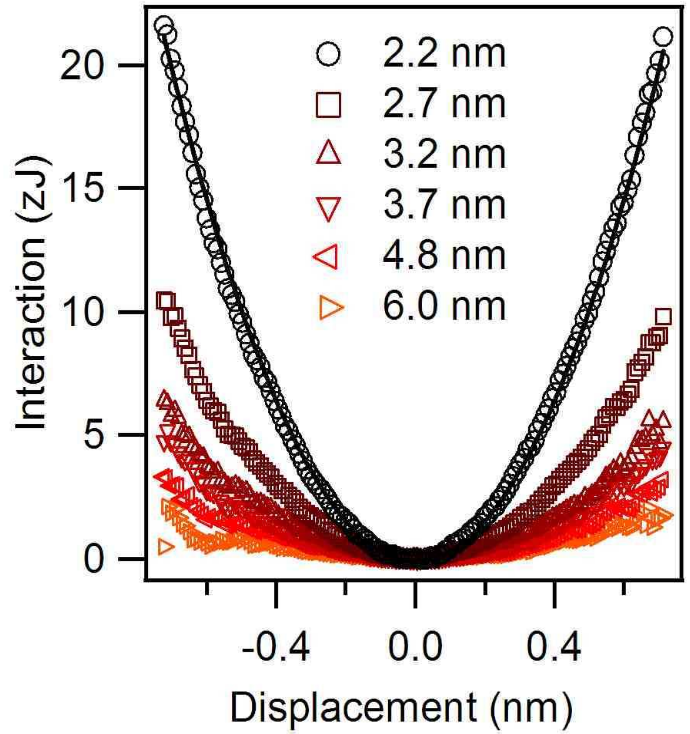
$$V_{ts} = V - V_C$$

V(u,z) total potential



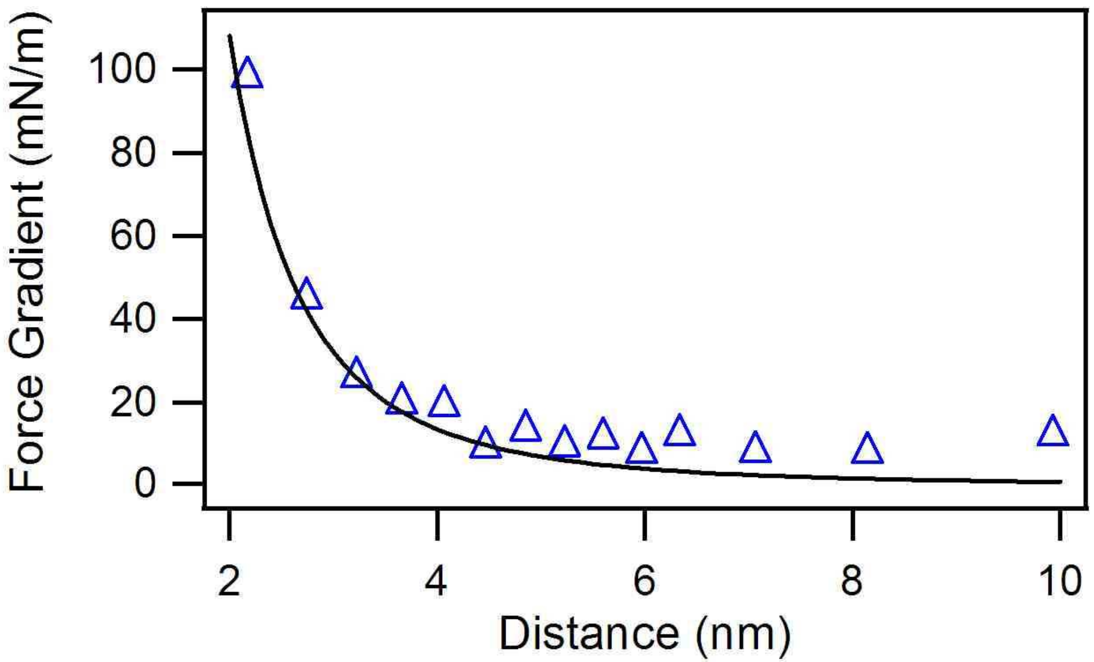
Anharmonic contribution
near the surface

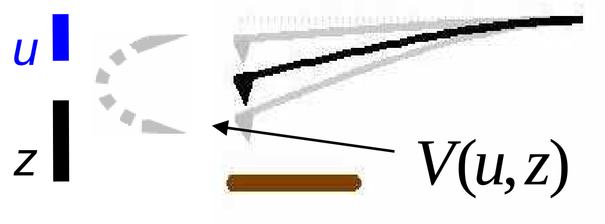
$V_{ts}(u,z)$ tip-sample
interaction potential



Interaction potential parabolic→
mass-spring approximation correct

Tip-sample forces gradient





Boltzmann probability distribution

△ $$\frac{\partial F_{ts}}{\partial z} = -\frac{\partial^2 V_{ts}}{\partial s^2}$$

Frequency shift method

— $$\frac{\partial F_{ts}}{\partial z} = \frac{HR}{3(z - z_0)^3}$$

$HR = 3 \cdot 10^{-27}$ J m

from the frequency shift data fitting

- Introduction
- Free cantilever thermal vibrations
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  - Frequency shift
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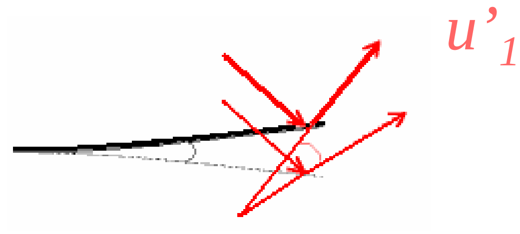
$$\frac{1}{2}K_B T = \frac{1}{2}k\langle u^2 \rangle$$

Equipartition Theorem

k free cantilever spring constant

$\langle u^2 \rangle$ cantilever mean square deflection



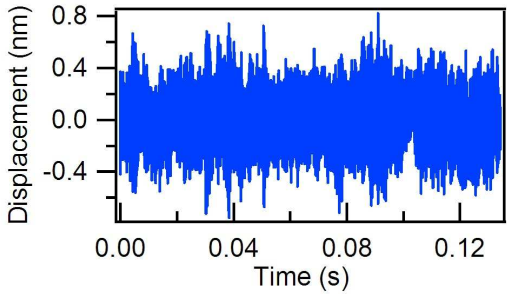
The first mode mean square virtual deflection $\langle u'^2_1 \rangle$ and then the effective spring constant k^* are evaluated at different tip-sample distances z

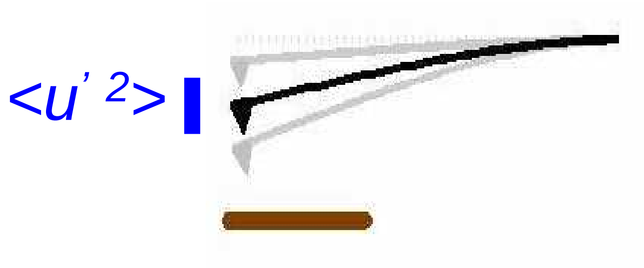
$$k^* = 0.82\frac{K_B T}{\langle u'^2_1 \rangle}$$

$$\frac{\partial F_{ts}}{\partial z} = k - k^*$$

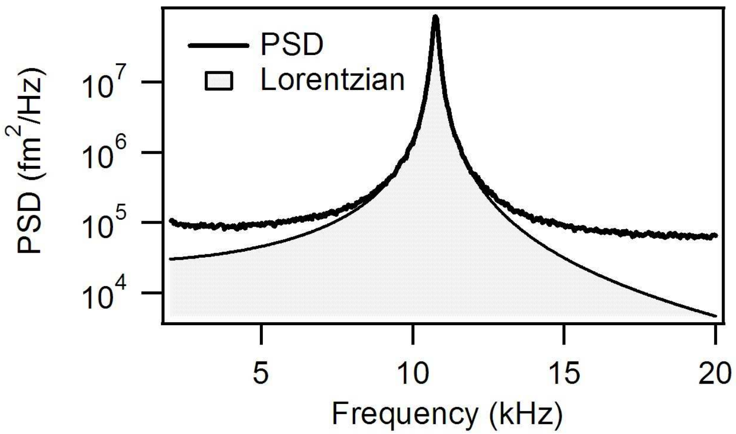
H.J.Butt *et al.*, *Nanotecnology* **6**, 1 (1995)
R.Lévy *et al.*, *Nanotechnology* **13**, 33 (2002)

Time domain → electronic noise adds to the cantilever virtual deflection





Frequency domain → thermal oscillations isolated from white noise background



$\langle u'^{2}_{1} \rangle$ equals the integral of the power spectrum of the thermal fluctuation alone (Lorentzian function) →

fit minus white-noise background

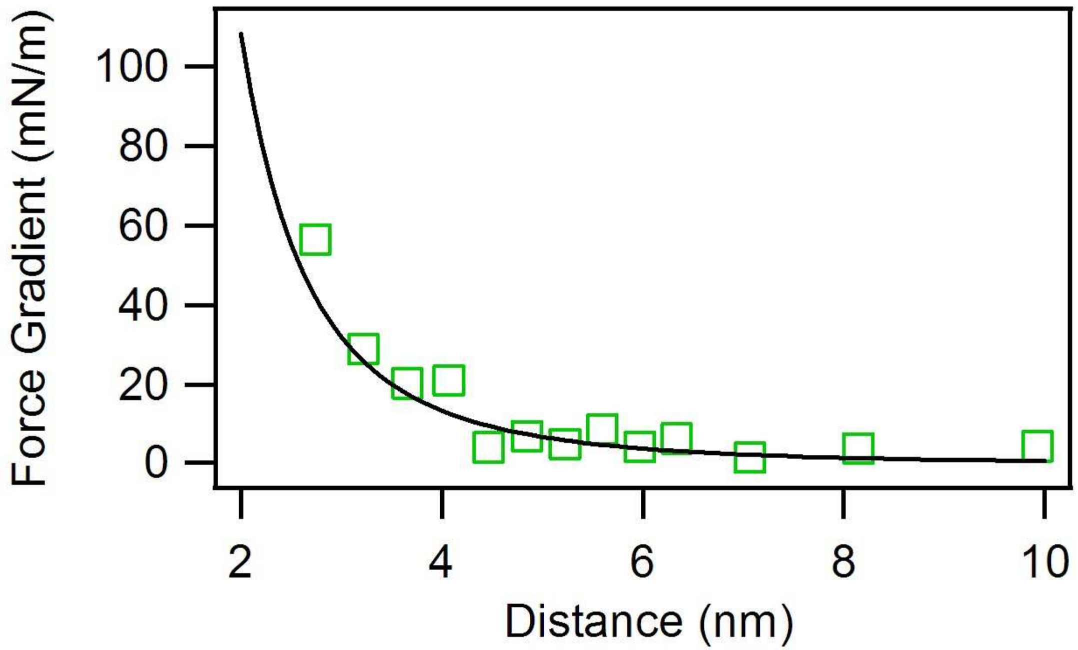
J.L.Hutter *et al.*, *Rev. Sci. Instrum.* **64**, 17 (1993)

Mean-square displacement from P.S.D.

$$k^* = 0.82\frac{K_B T}{\langle u'^2_1 \rangle}$$

$$\frac{\partial F_{ts}}{\partial z} = k - k^*$$

Tip-sample forces gradient



Frequency shift method

$$\frac{\partial F_{ts}}{\partial z} = \frac{HR}{3(z - z_0)^3}$$

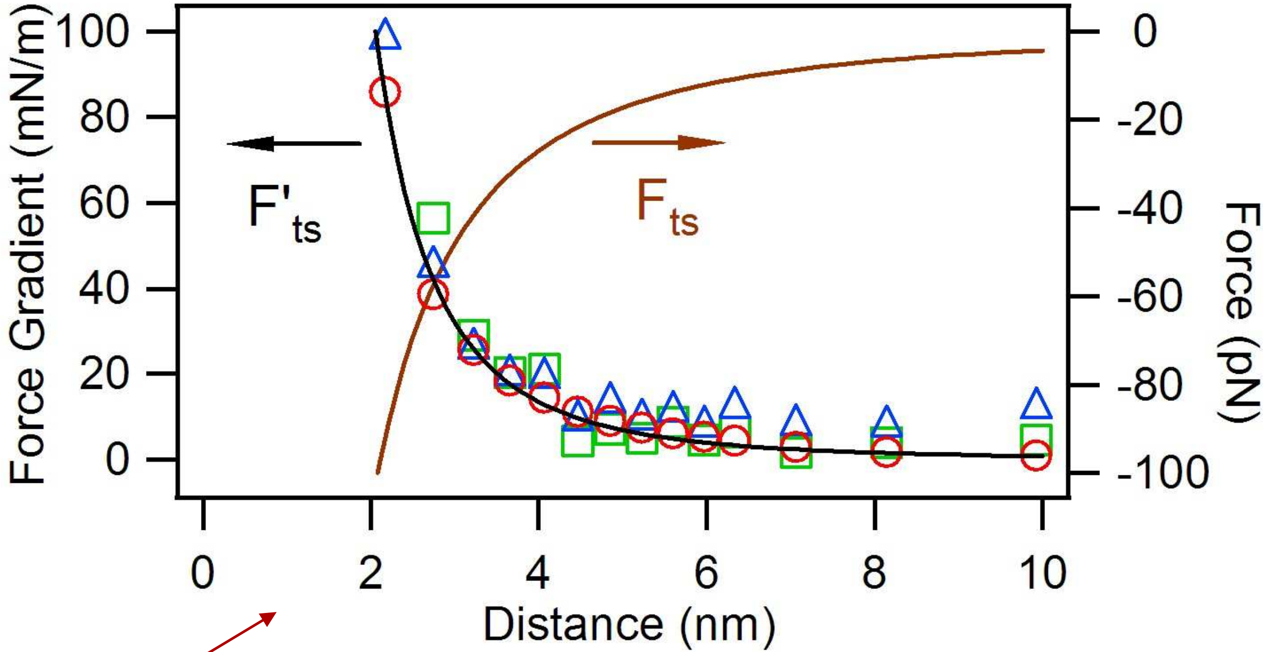
$HR = 3 \cdot 10^{-27}$ J m

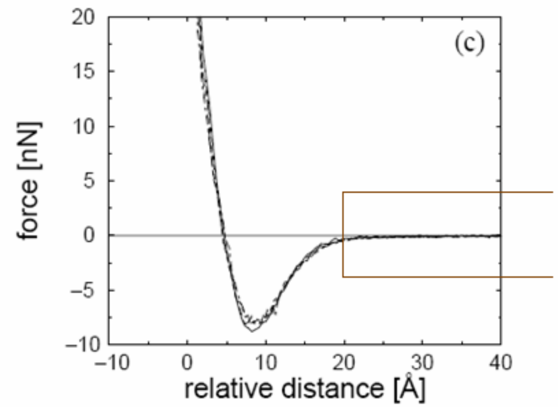
from the frequency shift data fitting

Thermal oscillations

$$F_{ts} = -\frac{HR}{6(z-z_0)^2}$$

$HR = 3 \cdot 10^{-27}$ J m





From nN to pN range

1. Frequency shift ○
2. Potential △
3. Mean square displacement □

Long-range interaction forces of the order of tens of piconewton

Intermittent contact → Short-range forces

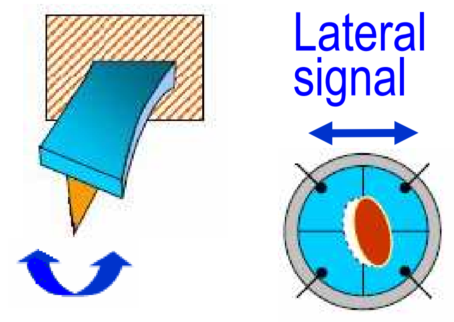
G.Malegori *et al.*, *J. Vac. Sci. Techn.* in press (2010)

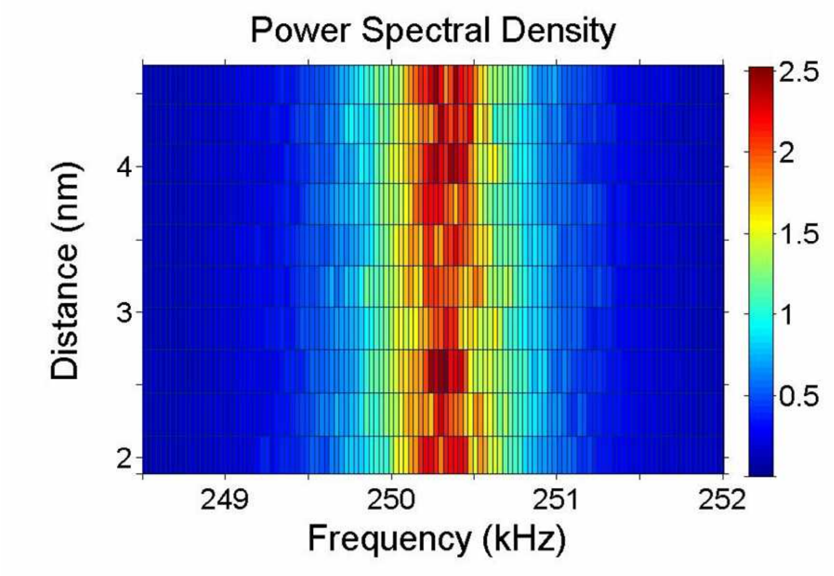
H.Hoelscher *et al.*, *Phys. Rev.Lett.* **83**, 4780 (1999)

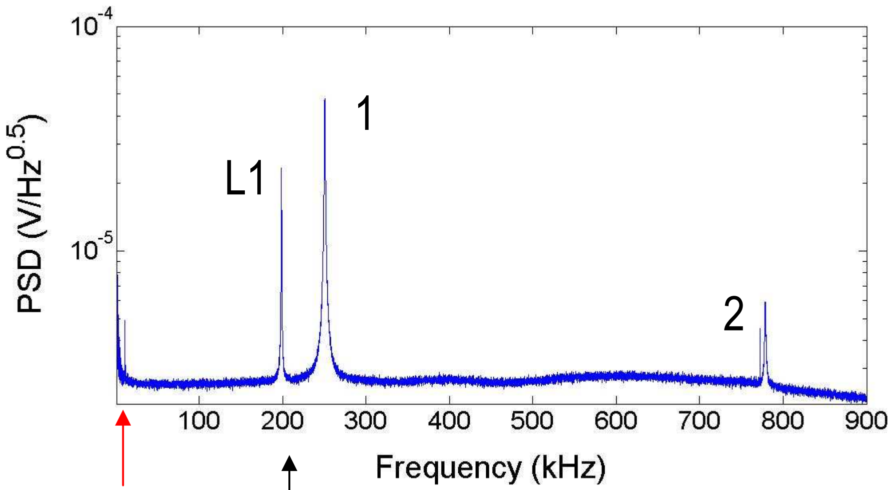
Thermal fluctuations of the AFM cantilever can be exploited to probe the tip-sample interaction

- One temporal trace of the cantilever thermal motion
  Three different approaches simultaneously used to measure the tip-sample interaction
- The distance dependence and the magnitude of the force observed are in good agreement with a non retarded van der Waals interaction
- Frequency shift method: mass loading effects on the second eigenmode
- Drawback: due to jump to contact only long-range forces are analyzed

- Investigation of other surfaces
- Measurements in liquids
- Automates the spectroscopy
- Torsional modes analysis







Lateral bending mode (as flexural interchanging cantilever width and thickness) [*]

[*] Espinoza-Beltran *et al.*, *New J. Phys.* **11**, 083034 (2009)

Acknowledgements

- Vittorio Spreafico, Stefano Prato
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  gabriele@dmf.unicatt.it
  http://www.dmf.unicatt.it/elphos

- Fulvio Parmigiani
  Sincrotrone Trieste
  Università degli Studi di Trieste
  Dipartimento di Fisica, Basovizza, Trieste, Italy

Frequency shift method

The resonance frequency of free cantilever is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m^*}}$$

k spring constant, *m** effective mass

The tip-sample interaction F_{ts} changes the resonance frequency.
For small amplitude oscillations

$$k^* = k - \frac{\partial F_{ts}}{\partial z} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k^*}{m^*}}$$

k* effective spring constant

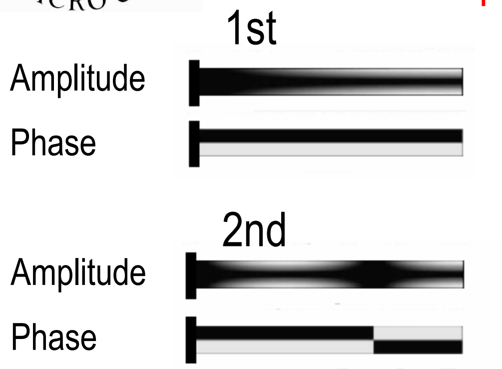
The frequency shift is directly related to the tip-sample force gradient (exact solution)

$$f = f_0 + \Delta f$$

$$\frac{\partial F_{ts}}{\partial z} = k - k^* = k\left(1 - \frac{f^2}{f_0^2}\right)$$

Y.Martin *et al*., *J. Appl. Phys.* **61**, 4723 (1987)
F.J.Giessibl, *Rev. Mod. Phys.* **75**, 949 (2003)

Free cantilever's torsional modes



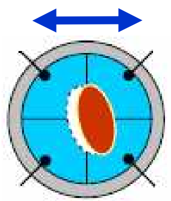
Material properties

Geometrical parameters

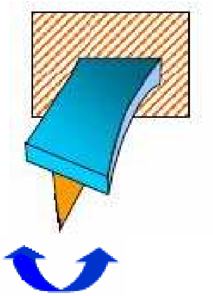
$$\frac{f_n^T}{f^L} = \frac{(2n-1)\pi\sqrt{6}}{\alpha_1^2}\sqrt{\frac{1}{1+\nu}}\frac{L}{w}$$

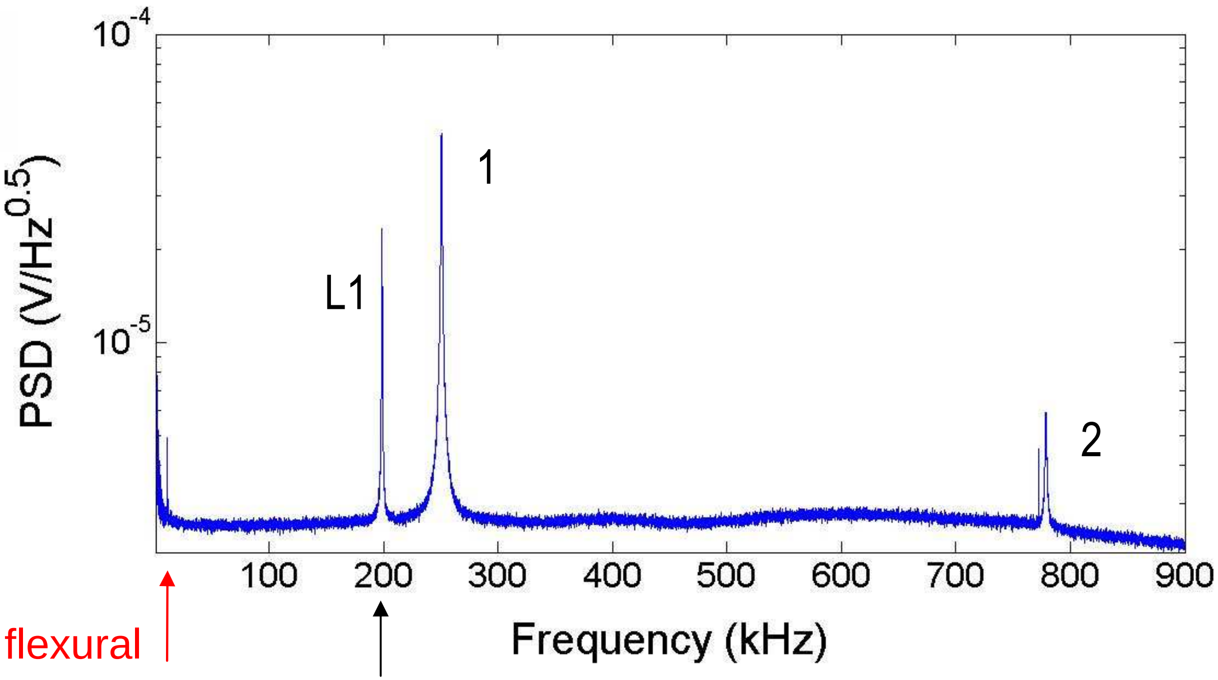
Boundary conditions

Torsional mode	1	2
f_n (kHz)	250	779
f_n / f_1 (exp.)	23	72
f_n / f_1 (th. [1])	24.71	74.13



Lateral signal





1st flexural

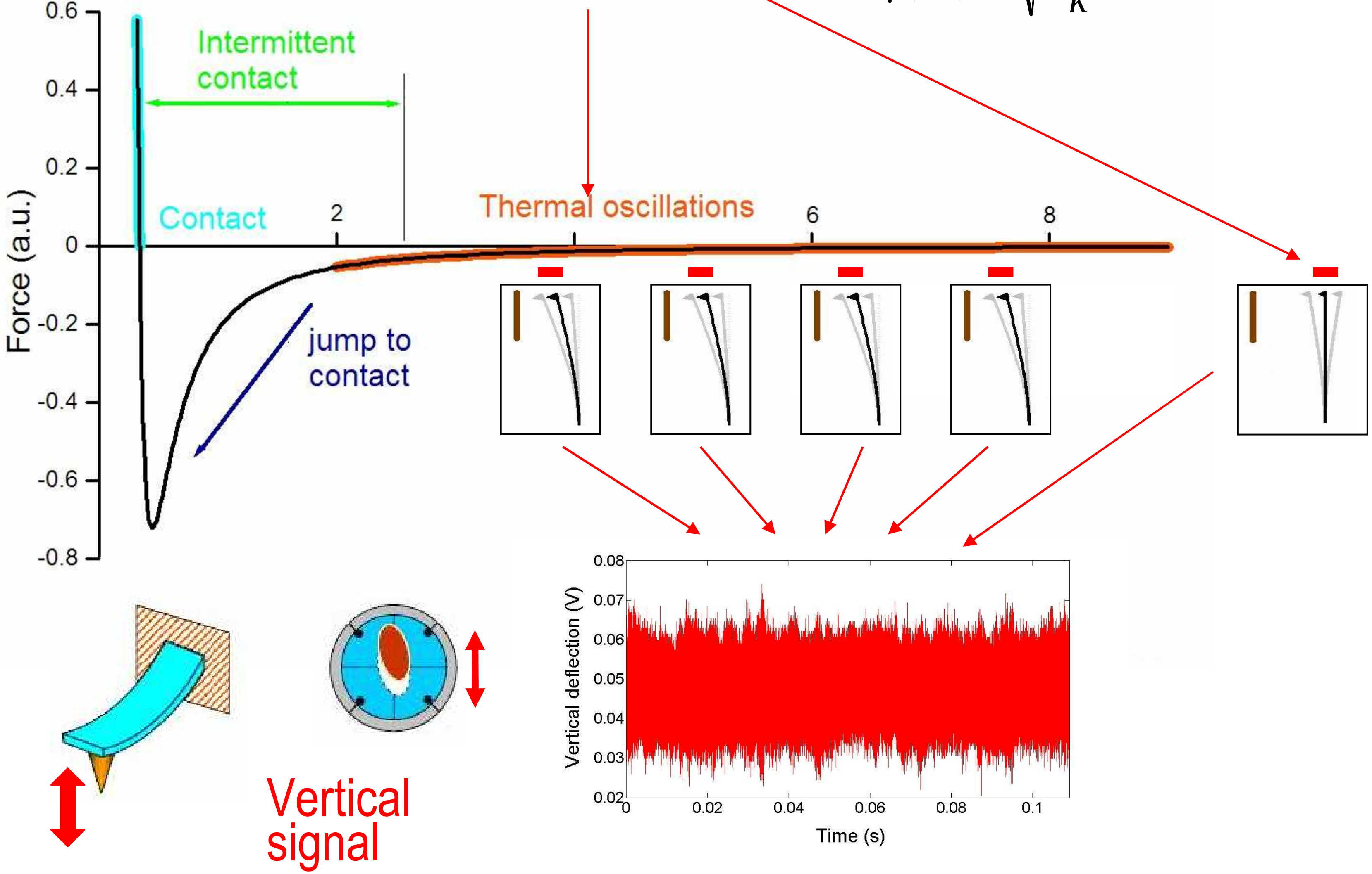
Lateral bending mode (as flexural interchanging cantilever width and thickness) [2]

[1] H.J. Van Eysden *et al.*, *J. Appl. Phys.* **101**, 044908 (2007)

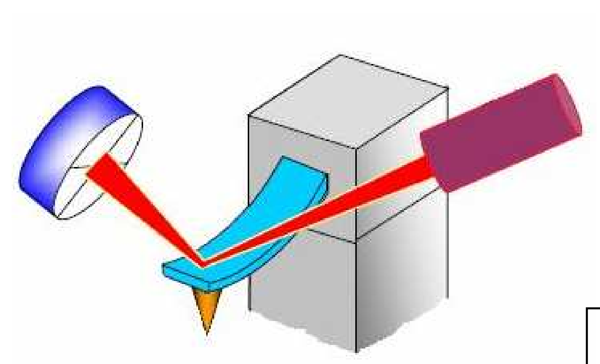
[2] Espinoza-Beltran *et al.*, *New J. Phys.* **11**, 083034 (2009)

Tip-sample interaction

Thermal vibrations amplitude $\sqrt{\langle u^2 \rangle} = \sqrt{\frac{k_B T}{k}} < 0.2$ nm

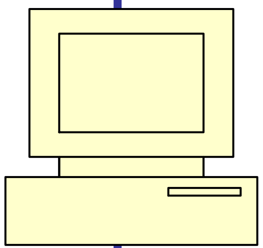


Experimental setup



Acquisition system

- Bandwidth: > 1 MHz
- Buffer memory: 128MS
- Sensitivity: 50-200 nm/V

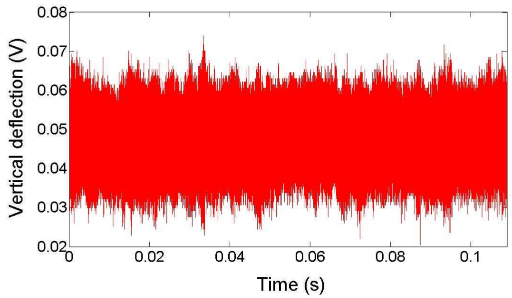


Cantilever tilt 15° → Senitivity correction [*]

Time signal

Vertical → flexion

Lateral → torsion

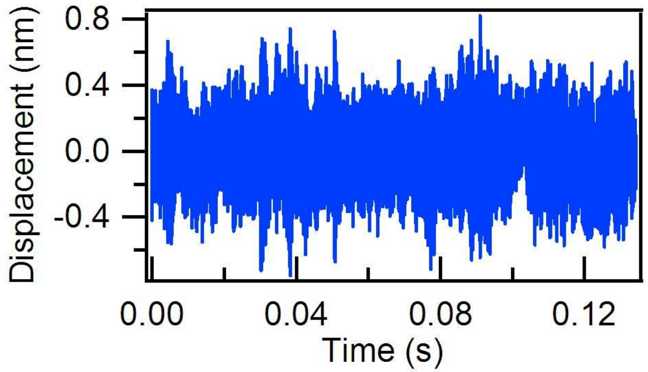


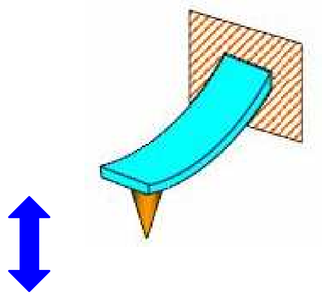
Off-line analysys

- Frequency shift method
- Potential from Boltzmann distribution
- Mean-square displacement from power spectral density

[*] J.L.Hutter, *Langmuir* **21**, 2630 (2005)

Boltzmann probability distribution



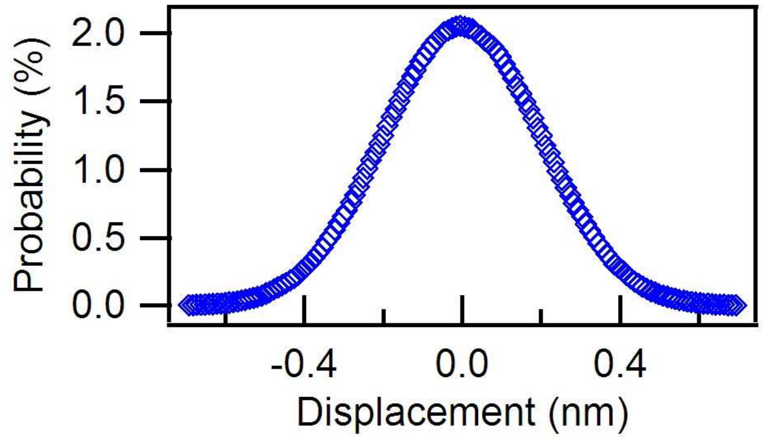


- *p(u,z)* probability of observing the tip at a deflection *u* from the equilibrium position/tip-surface distance *z* →
- number of count at the deflection *u* divided by the total number of count

Boltzmann probability distribution

$$p(u,z) = p_0 e^{-\frac{V(u,z)}{K_B T}}$$

V(u,z) position dependent total potential



W.F. Heinz *et al.*, *J. Phys. Chem. B* **104**, 622 (2000)
D.O.Koralek *et al.*, *Appl. Phys. Lett.* **76**, 2952 (2000)

Potential from Boltzmann distribution

Boltzmann probability distribution

$$p(u) = p_0 e^{-\frac{V(u)}{K_B T}}$$

- p_0 normalization constant
- $V(u)$ position dependent potential
- $p(u)$ probability of observing the tip at a deflection u from the equilibrium position/tip-surface distance z

Inverting the relation gives the total potential $V(u)$

$$V(u) = -K_B T \ln \frac{p(u)}{p_0}$$

$$V(u,z) = V_C(u,z) + V_{ts}(u,z)$$

- V_c free cantilever potential
- V_{ts} tip-sample interaction potential

W.F. Heinz *et al.*, *J. Phys. Chem. B* **104**, 622 (2000)
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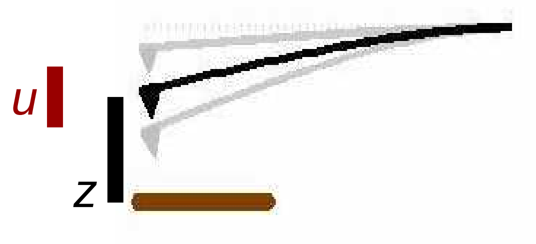
$$V(u,z) = V_C(u,z) + V_{ts}(u,z)$$

$$V_C = \frac{1}{2} k u^2$$

- V_c free cantilever potential
- V_{ts} tip-sample interaction potential

Small oscillation amplitude *u* ($\partial F_{ts} / \partial z$ constant) → harmonic potential

$$V = \frac{1}{2} k^* u^2 \qquad k^* = k - \frac{\partial F_{ts}}{\partial z}$$



Tip-sample force gradient

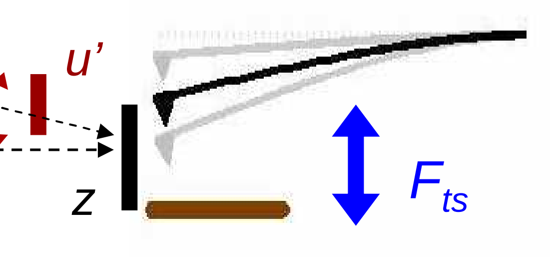
$$\frac{\partial F_{ts}}{\partial z} = -\frac{\partial^2 V_{ts}}{\partial s^2}$$

W.F. Heinz *et al.*, *J. Phys. Chem. B* **104**, 622 (2000)
D.O.Koralek *et al.*, *Appl. Phys. Lett.* **76**, 2952 (2000)

Interacting cantilever

Cantilever near the surface → F_{ts} interacting force

$$m^* \ddot{u} + \gamma \dot{u} + ku = F_{ts}(z) = F_{ts}(z_0) + \frac{\partial F_{ts}(z_0)}{\partial z} u$$



- The constant force $F_{ts}(z_0)$ only displaces the equilibrium position z_0
- The force derivative $\partial F_{ts} / \partial z$ influences the cantilever oscillations

Small amplitude oscillations → the force gradient $\partial F_{ts} / \partial z$ is constant during the cantilever oscillations *u'* around the new equilibrium position

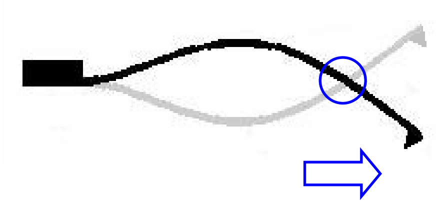
$$m^* \ddot{u}' + \gamma \dot{u}' + k^* u' = 0 \qquad k^* = k - \frac{\partial F_{ts}}{\partial z}$$

The tip-sample force gradient is directly related to the effective spring constant k^*

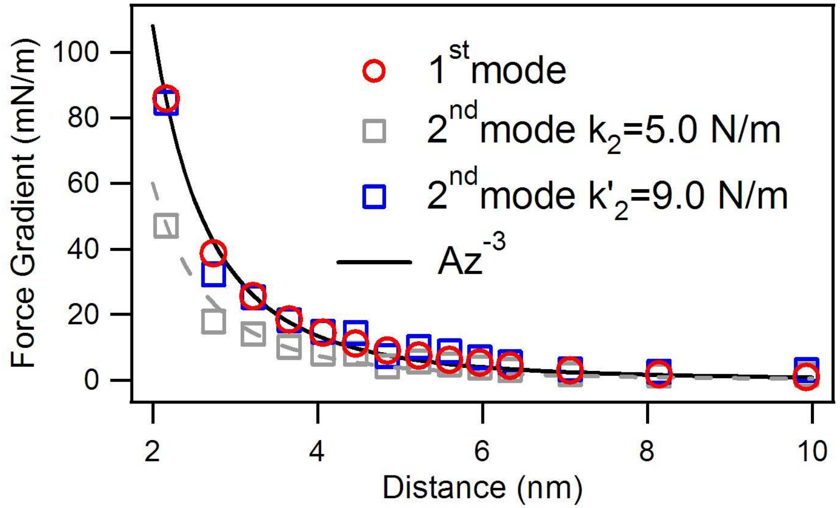
V.L.Mironov *Fundamentals of SPM* (2004)
F.J.Giessibl, *Rev. Mod. Phys.* **75**, 949 (2003)

Tip mass loading

1st: no significant effect



2nd: the node shifts toward the free end,
the cantilever effective length reduces
the equivalent stiffness increases



- A tip mass that is 10% of the cantilever mass nearly doubles the second mode's equivalent stiffness. [1]
- Sader method [2] is accurate only for the lower modes.

k =0.12 N/m [2]	k_2 Sader meth.	k'_2 fit to k_{ts}	Th. no tip [1]	Th. tip 10% CL [1]
k_2 (N/m)	5.0	**9.0**		
k_2/k	42	**75**	40.2	**74.9**

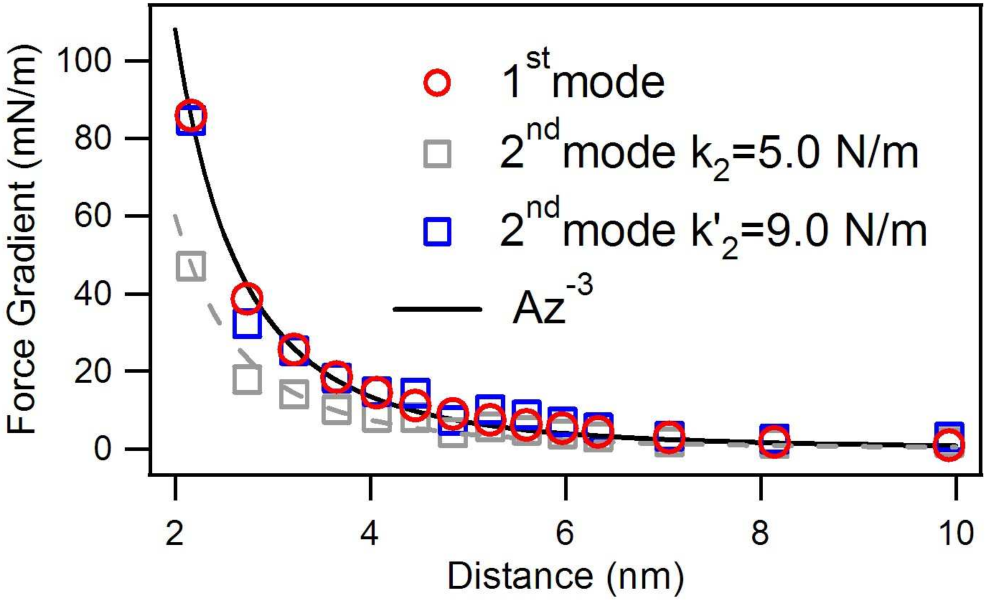
[1] J.Melcher *et al.*, *Appl. Phys. Lett.* **91**, 053101 (2007)
Y.Sugimoto *et al.*, *Appl. Phys. Lett.* **91**, 093120 (2007)

[2] J.E.Sader *et al.*, *Rev. Sci. Ins.* **70**, 3967 (1999)

Tip mass loading

- The tip mass doesn't influence significantly the equivalent stiffness of the first eigenmode but can have dramatic effect on the equivalent stiffness of the higher eigenmodes.

- A tip mass that is 10% of the cantilever mass nearly doubles the second mode's equivalent stiffness. [1]
- Sader method [2] is accurate only for the lower modes.



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