# Dynamics of random dipoles : chaos *vs* ferromagnetism

F. Borgonovi[1, 2] and G. L. Celardo[3]
[1]*Dipartimento di Matematica e Fisica, Università Cattolica, via Musei 41, 25121 Brescia, Italy*
[2]*I.N.F.N., Sezione di Pavia, Italy*
[3]*Dept of Physics, Tulane University, Newo Orleans USA*
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The microcanonical dynamics of an ensemble of random magnetic dipoles in a needle has been investigated. Analyzing magnetic reversal times, a transition between a chaotic paramagnetic phase and an integrable ferromagnetic phase has been numerically found. In particular, a simple criterium for transition has been formulated. Close to the transition point the statistics of average magnetic reversal times and fluctuations have been studied and critical exponents numerically given.

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## I. INTRODUCTION

A truly comprehensive understanding of magnetism at the nanoscale is still lacking and has important consequences in the technology of memory and information processing devises.

Many unsolved problems about magnetic properties of diluted spin systems attracted recently great attention. Among the open problems there is the emergence of ferromagnetism in doped diluted systems[1], where the Curie temperatures can be as high as 300 K, and a deep theoretical understanding of the magnetic properties of dilute dipole systems (spin glass transition, ferromagnetic and anti-ferromagnetic transitions).

Here we will concentrate on randomly arranged dilute classical dipoles, which are called dipole glasses. Many results in literature, sometimes controversial, exist on such kind of systems. Magnetic properties of dipole-dipole interacting spins are particularly difficult to study due to many factors: long range nature of the interaction, anisotropy and frustration. Long range and anisotropy can induce ergodicity breaking[2] in a system. Breaking of ergodicity, a concept introduced by Palmer[2], and recently found explicitly [3, 4] in a class of long-ranged anisotropic spin systems, is a key word to understanding phase transitions too, even if it should not be confused with breaking of symmetry[5]. Speaking loosely, few constants of motion, such as the energy, or the angular momentum, in a particular geometry, produce a separation of the allowable phase space in two or more subspace over which the motion is constrained. In Ref.[3] the energy at which the separation occurs has been calculated explicitly for an anisotropic 1–D classical Heisenberg systems. In that case both the anisotropy and the long ranged nature[6] of the inter-spin interaction, are essential ingredients in order to have breaking of ergodicity[7]. On the other hand, frustration, that is the impossibility to attain a global minimal energy minimizing locally the interactions, induces a dependence of the ferromagnetic and anti-ferromagnetic properties on the lattice geometry [8].

Many results concern also the so-called Ising dipole glass, where Ising simply means uni-axial. To quote but a few: spin glass transition for high concentration, using Monte Carlo simulation[9, 10], mean field spin glass transition at low concentration depending on the lattice geometry[11], no spin glass transition for low concentration using Wang-Landau Monte Carlo simulation [12] or the recent spin glass transition at non zero temperature from extensive numerical simulation[13].

In this paper we will focus our analysis on dipole glass of freely rotating classical dipoles. First of all the dipole glass is a typical example of very frustrated system [14–16], so that different ground state configurations can exist depending on the geometry and the spin concentration. Results in the canonical ensemble typically consider a mean field approach, and it is common lore that the random positions of the dipoles induce magnetic field fluctuations. These fluctuations do not vanish at $T \to 0$, unlike thermal fluctuations, and tend to suppress magnetic order even at $T = 0$[15, 16]. So, magnetic order, is expected to happen only for high impurity concentration (and small temperature) [16–18]. Mean field theories consider only the equilibrium properties and do not take into account the time needed to reach the equilibrium situation and finite size effects. On the other side the question of how long a metastable state can last is a major issue in determining the magnetic properties of a system.

In this paper we study the microcanonical dynamics, reserving the study of the influence of a thermal bath for further investigations. We analyze the microcanonical dynamics of dipoles put at the vertexes of a cubic lattice (so that their relative distance cannot be smaller than the lattice size), only on the basis of the Landau-Lifshitz-Gilbert equations of motion. 3–D dipole-dipole interacting systems can be realized quite easily in laboratory, for instance doping a non magnetic media with paramagnetic ions, weakly integrating with the lattice and with a relative inter-dipole distance sufficiently large in order to neglect Heisenberg interaction (therefore with a low concentration).

Anticipating few of the results, we have found that taking into account a typical experimental situation with needle-shaped sample, a further constant of motion appears that induce another kind of "phase" transition related to invariant tori, which separate the allowable phase in many disconnected regions. In this particular case, the ergodicity breaking is not due to an increase of energy, but to an increase of perturbation, which means the tendency to a transformation from a needle shape (quasi 1–D system) to a cubic shape (3–D shape). In a sense, these results are more akin to the standard perturbation theory in classical dynamical systems[19, 20], re-interpreted in the light of phase transitions induced by magnetic reversal times [21].

In the future we intend to study the same system in contact with a thermal bath. In this case the presence of the ergodicity breaking found in [3, 4] should influence the magnetic reversal times. In the microcanonical case, the presence of this ergodicity breaking is hidden by quasi-integrability of motion.

## II. THE CLASSICAL MODEL AND THE PERTURBATIVE APPROACH

Let us consider a system of $N$ classical dipoles $\vec{\mu}_i$ randomly put at the nodes of a 3–D gridded box $R \times R \times L$, with $L \gg R$, and low concentration $\delta \ll 1$, as indicated in Fig. 1,

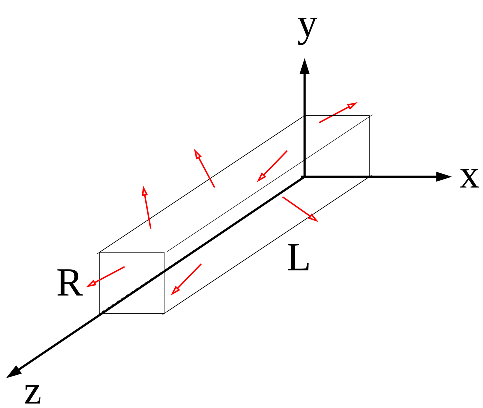


FIG. 1: Needle geometry. The classical dipoles are put in a random way on the vertexes of a cubic lattice of size $a$. $R$ and $L$ are given in units of the lattice size $a$.

From the physical point of view it represents a dilute system of paramagnetic ions in a non magnetic bulk, with a concentration $\delta = N/N_s$ where $N_s = R^2 L$ is the number of allowable sites in the 3–D lattice. As explained above, such a system can be realized in laboratory, doping a non–magnetic system having a cubic lattice with paramagnetic impurities. In the last decade, a lot of experimental and theoretical results have been collected for doped $TiO_2$ and other[1]. If the dipoles weakly interact with the lattice and if their average distance is much greater than the Bohr radius, we can simply neglect the Heisenberg (exchange) interaction and represent their mutual interaction and dynamics with a pure dipole-dipole interaction energy:

$$E = \frac{\mu_0 \mu^2}{4\pi a^3} \sum_{i=1}^{N} \sum_{j>i} \frac{1}{|r_{ij}|^3} \left[ \vec{S}_i \cdot \vec{S}_j - 3(\vec{S}_i \cdot \hat{r}_{ij})(\vec{S}_j \cdot \hat{r}_{ij}) \right]. \tag{1}$$

Here $\vec{S}$ is the dimensionless spin vector

$$\vec{S} \cdot \vec{S} = 1, \tag{2}$$

$\mu$ is the magnetic moment of the paramagnetic doping ions and $r_{ij}$ is the distance between the i-th and the j-th spin in units of the lattice spacing $a$.

The dynamics is described by the Landau-Lifshitz-Gilbert equations of motion:

$$\frac{d}{dt}\vec{\mu}_k = \gamma \vec{\mu}_k \times \frac{\delta E}{\delta \vec{\mu}_k}, \tag{3}$$

where $\vec{\mu}_k = \mu \vec{S}_k$ and $\gamma$ is the gyromagnetic ratio. They can be rewritten in the dimensionless form,

$$\frac{d}{d\tau}\vec{S}_k = \vec{S}_k \times \frac{\delta E_0}{\delta \vec{S}_k}, \tag{4}$$

where the following dimensionless quantities have been introduced:

$$\begin{aligned} E_0 &= E\frac{4\pi a^3}{\mu_0 \mu^2} \\ \tau &= \omega t, \quad \text{with} \quad \omega = \frac{\gamma \mu \mu_0}{4\pi a^3}. \end{aligned} \tag{5}$$

To fix ideas, let us put some numbers: for $\mu = \mu_B$ (Bohr magneton) and $a = 10^{-10}$ m, one has $\omega = 81.5$ Hz.

The system of equations considered above conserves the energy (1) and the squared modulii of the spins (2).

The diluted doped quasi 1–D system can be magnetized with a strong magnetic field directed along $L$, the longest axis ($z$-axis). The questions we would like to answer is the following: What is the dependence of the average magnetic reversal time and its fluctuations on the system parameters?

The relevant parameters to take into account are the concentration $\delta$ of paramagnetic ions and the aspect ratio $\epsilon = R/L$. In principle, due to the long-ranged nature of the dipole interaction, one could ask whether there are effects dependent on both the system size and the number of doping spins $N$, even if in quasi 1–D systems, the dipole interaction can be treated as a short range interaction.

From the point of view of the equations of motion (4), if the $N$ dipoles are lying along a straight line ($R = 0 \Rightarrow \epsilon = 0$), there is a further constant of motion, i.e. $M_z = (1/N)\sum_k S_k^z$. Therefore, for a 1–D system, the answer to the first question above is very simple : a state with any initial magnetization $M_z(0) \neq 0$ will keep the initial magnetization forever. The natural question thus

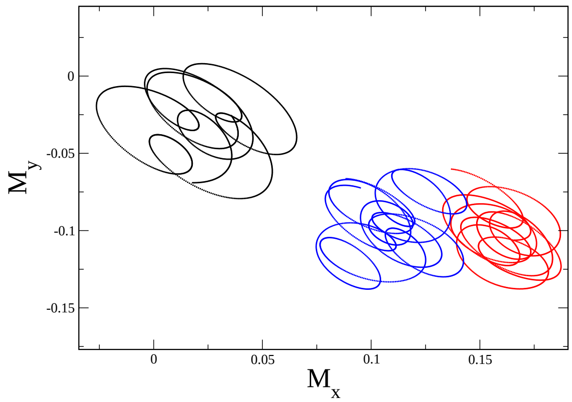


FIG. 2: Three different trajectories, for $R = 4$, $L = 4000$, $\delta = 10^{-3}$, $N = 64$, in the integrable case. Initially spins are chosen with random components on the unit sphere.

becomes: what happens for $\epsilon \neq 0$? Will a magnetized state demagnetize and how much time it takes to?

The classical dynamical picture can be simplified adopting a perturbative approach, namely approximating the distance between two spins as follows:

$$\hat{r}_{ij} \simeq \hat{z} + \epsilon(\cos\phi_{ij}\hat{x} + \sin\phi_{ij}\hat{y}), \tag{6}$$

where $\hat{x}, \hat{y}, \hat{z}$ are the unit versors and $\phi_{ij}$ are the azimuthal angles with respect the $z$-axis. The energy (1), to first order in $\epsilon$, becomes: $E_0 = H_0 + \epsilon V$, where $H_0$ is the energy part that conserve $M_z$, and $V$ is the perturbation,

$$\begin{aligned} H_0 &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\frac{1}{|r_{ij}|^3}\left[S_i^x S_j^x + S_i^y S_j^y - 2S_i^z S_j^z,\right] \\ V &= -3\sum_{i=1}^{N}\sum_{j\neq i}\frac{1}{|r_{ij}|^3}\left[\cos\phi_{ij} S_i^z S_j^x + \sin\phi_{ij} S_i^z S_j^y\right]. \end{aligned} \tag{7}$$

The equations of motion for the macroscopic variables, $M_{x,y,z}$ can be written as,

$$\begin{aligned} \frac{dM_z}{d\tau} &= \frac{3\epsilon}{N}\sum_k\sum_{i\neq k}\frac{1}{|r_{ik}|^3}S_i^z\left(S_k^y\cos\phi_{ik} - S_k^x\sin\phi_{ik}\right) \\ \frac{dM_y}{d\tau} &= \frac{3}{N}\sum_k\sum_{i\neq k}\frac{1}{|r_{ik}|^3}\{S_i^z S_k^x + \\ &\epsilon\left[S_k^x S_i^y\sin\phi_{ik} + (S_k^x S_i^x - S_k^z S_i^z)\cos\phi_{ik}\right]\} \\ \frac{dM_x}{d\tau} &= -\frac{3}{N}\sum_k\sum_{i\neq k}\frac{1}{|r_{ik}|^3}\{S_i^z S_k^y + \\ &\epsilon\left[S_k^x S_i^y\cos\phi_{ik} + (S_k^y S_i^y - S_k^z S_i^z)\sin\phi_{ik}\right]\}, \end{aligned} \tag{8}$$

and, in particular, for $\epsilon = 0$ they assume the suggestive form:

$$\begin{aligned} \frac{dM_z}{d\tau} &= 0 \\ \frac{dM_y}{d\tau} &= \frac{1}{N}\sum_k \omega_k S_k^x \\ \frac{dM_x}{d\tau} &= -\frac{1}{N}\sum_k \omega_k S_k^y, \end{aligned} \tag{9}$$

having defined, the average "local" frequencies:

$$\omega_k = 3\sum_{i\neq k}\frac{1}{|r_{ik}|^3}S_i^z. \tag{10}$$

These equations describe a kind of rotation in the plane perpendicular to the $z$–magnetization (which is a constant of motion). Therefore one could expect that for $\epsilon \ll 1$ a rotational-like motion about the $z$–axis persists, while $M_z$ remains a quasi constant of motion. This is what can be observed for instance by a direct inspection of the trajectories of the macroscopic vector $\vec{M}$, in the plane $x, y$, see Fig. 2, where few selected trajectories has been iterated in time, for $\epsilon = 10^{-3}$. Quite naturally, on increasing the perturbation strength $\epsilon$, one could expect that the invariant tori $M_z = const$ will be broken, and, eventually, a stochastic motion of the macroscopic variable $M_z$ will appear. In the next Section we will study the survival of invariant tori, under the dimensional perturbation $\epsilon > 0$.

## III. THE CHAOTIC–PARAMAGNETIC AND THE INTEGRABLE–FERROMAGNETIC PHASES

The dynamical behavior of the system can be characterized by a "regular region" $\epsilon < \epsilon_{cr}$ in which the magnetization $M_z(\tau)$ is bounded in a small interval $\delta M_z$, while, for $\epsilon > \epsilon_{cr}$, $M_z(\tau)$ quickly decays and after that it fluctuates around 0. To be more precise, the transition across $\epsilon_{cr}$ is smooth, namely there is a region of $\epsilon$ values in which the initial magnetization decay to some non zero constant when the time $\tau$ becomes large.

The critical value of the perturbation strength $\epsilon_{cr}$ can be obtained with the following hand-waving argument. Let us divide the 3–D box in $n = 1/\epsilon$ small cubic boxes of side $R$. If the impurities concentration $\delta$ is sufficiently small in order to have only one spin inside each $R$-side box then the system is approximately one dimensional and $M_z$ can be considered an approximate constant of motion. Otherwise, for large $\delta$, the system behaves like a 3–D system and $M_z$ can spread everywhere. In other words, the average distance between two spins (in units of lattice spacing) $d = (V/N)^{1/3} = (LR^2/N)^{1/3}$ should be larger than the transverse distance $R$ in order to behave like a 1–D systems (ferromagnetic). The critical $\epsilon$ is thus given by

$$\frac{LR^2}{N} \simeq R^3 \to \epsilon_{cr} \simeq \frac{1}{N} \tag{11}$$

Since $\delta = N/N_s$, and $N_s = R^2 L$, the condition in order to have ferromagnetism can also be written in the form

$$R^3 < \frac{1}{\delta}. \tag{12}$$

Since $R$ (the transverse distance in lattice units) must be greater than 1 it is clear that ferromagnetism due to quasi constant of motion can occur only in a very diluted system $\delta \ll 1$.

An example is shown in Fig. 3, where the dynamics of magnetization has been plot in the overcritical case ($\epsilon > \epsilon_{cr}$ Fig. 3a) and in the undercritical one ($\epsilon < \epsilon_{cr}$ Fig. 3b). Different trajectories, corresponding to different initial conditions $M_z(0)$ have been shown in different colors. As one can see, in the "paramagnetic" phase ($\epsilon > \epsilon_{cr}$) the magnetization first decays to zero and then it fluctuates randomly around zero. On the contrary, in the "ferromagnetic" phase ($\epsilon < \epsilon_{cr}$), it show a periodic behavior around the initial conditions.

This behavior is quite typical in the study of dynamical systems, where the increase of a suitable perturbative parameter is related to the breaking of invariant tori and to the emergence of chaotic motion [19, 20].

It is also remarkable to study the fluctuations around the asymptotic behavior: in the undercritical case (Fig. 3d) fluctuations are much smaller than in the overcritical case (Fig. 3c), roughly 10 times for this case, as can be seen comparing the width of the probability distribution functions in Fig. 3 c) and d).

The large fluctuations around the average values and in order to fit a possible experimental situation, suggests to perform an average over disorder, namely an ensemble of samples with different random configurations, initially magnetized along the $z$–axis.

The results for the ensemble average $\langle M_z \rangle$ are shown in Fig.4a, where the black line separates the two "phases" $\epsilon < \epsilon_{cr}$ and $\epsilon > \epsilon_{cr}$. The average magnetization in the undercritical regime reaches some equilibrium value different from zero after some initial decay, while in the overcritical regime it goes to zero in an algebraic way.

Ensemble fluctuations at the equilibrium are approximately independent of $\epsilon$ in the paramagnetic phase while in the ferromagnetic one are typically smaller and increasing as $\sqrt{\epsilon}$. They are presented in Fig. 4d), where

$$\Delta M_z^{eq} = \lim_{\tau \to \infty} \Delta M_z(\tau)$$

has been shown as a function of $\epsilon$. Each point on the plot corresponds to an ensemble of magnetized sticks, with the same concentration and number of spins (paramagnetic ions) but different aspect ratio $\epsilon$. It is quite remarkable that a critical value $\epsilon_{cr}$ exists, characterized by two different behaviors.

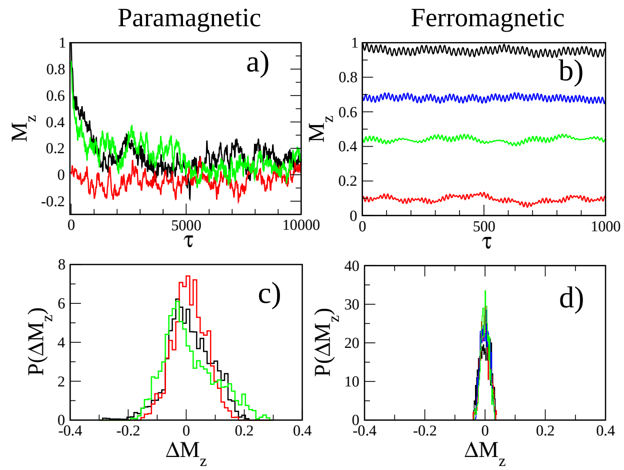


FIG. 3: Time behavior of the magnetization for different initial conditions in the overcritical case $\epsilon = 0.125$ (a) ($L = 160, R = 20$) and in the undercritical case $\epsilon = 10^{-3}$ (b) ($L = 4000, R = 4$). Other data, i.e. concentration $\delta = 10^{-3}$ and the number of spins $N = 64$ are the same for both cases. In c) and d) the probability distribution functions for the fluctuations $\Delta M_z = (\langle M_z^2 \rangle - \langle M_z \rangle^2)^{1/2}$ around the equilibrium value is shown for the data given respectively in a) and b).

Both the independence on the perturbation strength and the square root dependence on $\epsilon$ in the undercritical can be understood on the basis of classical dynamical theory. Breaking invariant tori with a perturbation strength $\epsilon$ corresponds to create stochastic layers between invariant tori whose size is proportional to $\sqrt{\epsilon}$[19, 20]. On the other side when the system is completely chaotic, since the variable $M_z$ is bounded, it can only occupy all the allowable stochastic region, and a further increasing of perturbation strength can not modify this size.

## IV. MAGNETIC REVERSAL TIME

Another important information can be obtained, in the paramagnetic phase, from the study of the magnetic reversal time $\tau_0$, defined by the first time at which $M_z(\tau_0) = 0$. Of course, even in this situation, we will consider the average magnetic reversal time $\langle \tau_0 \rangle$ where the mean is considered with respect to many different random realizations.

Having defined two "phases" characterized respectively by an infinite magnetic reversal time (integrable) and a finite one (chaotic), it can have some interest to study, if any, the critical exponent in which the magnetic reversal time approaches infinity when $\epsilon \to \epsilon_{cr}$. Indeed, analogous microcanonical investigation for long range spins systems gave indication of a power law divergence with an exponent dependent on the number of spins[21].

Before to show our results, let us say that a sufficiently

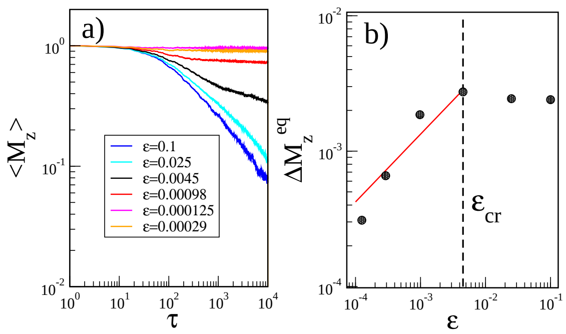


FIG. 4: a) Time behavior of the average magnetization for different values of the aspect ratio $\epsilon$, as indicated in the legend. b) Dependence of the equilibrium value of fluctuations as a function of $\epsilon$. Dashed vertical line indicates the critical value $\epsilon_{cr} = 1/N$. Red line indicates the dependence $\sqrt{\epsilon}$. Here is $N = 220$, $\delta = 10^{-3}$, and each line correspond to the average over 100 different random configurations. Initially we choose $S_i^z(0) = 1$, $i = 1, \dots, N$.

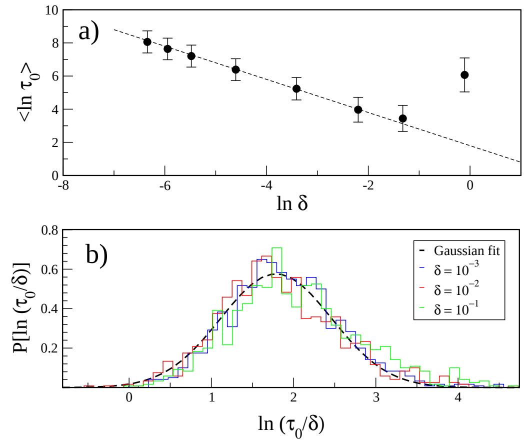


FIG. 5: a) Dependence of the average magnetic reversal time $\langle \ln \tau_0 \rangle$ as a function of concentration $\ln \delta$, for systems with $\epsilon = 0.1$ and $N = 72$. The average has been taken over an ensemble of 1000 different samples. Initially all samples have all spins aligned along the $z$–axis : $S_i^z(0) = 1, \quad i = 1, \dots N$. Dashed line represents $\langle \tau_0 \rangle \propto 1/\delta$. b) Probability distribution function for the variable $ln(\tau_0/\delta)$ in the paramagnetic phase for 3 different concentrations as indicated in the legend and same $\epsilon = R/L = 0.1$ and number of particles $N = 1/\epsilon_{cr} = 72$ in order to have the same distance $|\epsilon - \epsilon_{cr}|$ from the critical border. As a dashed line a Gaussian fit to the sum of the three distribution is also shown.

small concentration $\delta \ll 1$ is not a relevant parameter of our system, since magnetic reversal times scale in a very simple way with it. In order to prove that, let us observe that due to the particular quasi 1–D geometry, and the low concentration, closest dipoles give the major contributions to the energy. For instance, the configuration with all spins aligned along the $z$–axis will have an energy,

$$E' \propto \sum_{\langle i,j \rangle} \frac{1}{|r_{ij}|^3}$$

where the sum is taken over $N$ couples $\langle i, j \rangle$ of neighbor dipoles. In other words $E' \sim N/d^3 \sim N\delta$, where $d$ is the average distance between two dipoles.

On the other side the Landau-Lifshitz-Gilbert equations of motion are invariant under a simultaneous scaling of time and energy $\tau' = \tau/\delta$ and $E' = E\delta$ so that we will expect $\tau \propto 1/\delta$. This simple relation has been verified considering a system with the same aspect ratio $\epsilon$, and the same number of particles $N$ (so to have the same critical value $\epsilon_{cr}$) and changing concentration over 3 orders of magnitude. Results are presented in Fig. 5a, where $\langle \ln \tau_0 \rangle$ has been shown *vs* $\ln \delta$. To guide the eye a dashed line with slope $-1$ has been superimposed. The last point to the right corresponds to $\delta \sim 1$ where our approach is not valid.

The choice $\langle \ln \tau_0 \rangle$ instead of $\langle \tau_0 \rangle$ is not only for a good fitting but it is due to the log-normal character of the distribution of magnetic reversal times, as shown in Fig. 5b, where three different distributions for three different concentrations has been superimposed, using the rescaling variable $\ln \tau_0/\delta$. A remarkable fact from Fig. 5b is that, not only the average magnetic reversal times are inversely proportional to the concentration $\delta$, but the shape itself of the probability distribution function is the same.

The presence of a wide distribution of reversal times is due to the large number of possible spin configuration in a dilute random arrangement. Roughly speaking, the more the spins will be aligned along the $z$-direction, the more the system will resemble a 1–D system, implying long magnetic reversal times. In Fig. 6 we fix the concentration, $\delta$, and we considered $10^4$ different random configurations. For each configuration we computed the number of spin pairs with the same $z$ position, which is the numer of spins which are aligned perpendicularly to the $z$ direction. In Fig. 6 we show a clear correlation between the average magnetic reversal times and the number of spin pairs with the same $z$ position. Indeed the larger is the number of spin pairs with the same $z$ position, the lesses a system resembles a 1–D system.

Since the concentration is not a relevant parameter, we fix it, and we consider in which way magnetic reversal times behave on approaching the critical border. There are essentially three ways to approach the critical border keeping fixed the concentration. The first one is keeping fixed the aspect ratio $\epsilon$ and changing the number of spins. A second way consists of keeping fixed the number of spins but changing $R$ and $L$ (and so the aspect ratio $\epsilon$). A third way is to keep fixed $R$ and changing both the

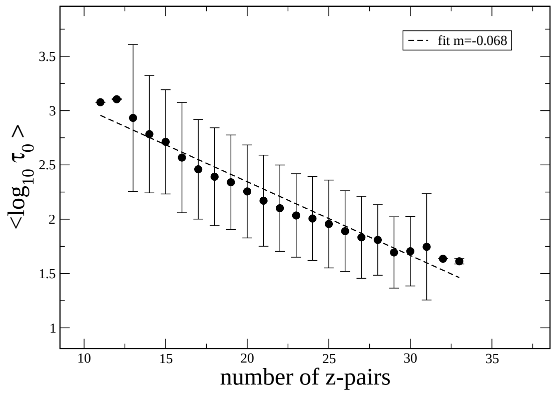


FIG. 6: Dependence of the average magnetic reversal time $\langle \log_{10} \tau_0 \rangle$ as a function of number of spin pairs with the same $z$ position, for systems with $\delta = 0.1$, $R = 3$, $L = 71$ and and $N = 63$. In this case $\epsilon = R/L = 0.042$ while $\epsilon_{cr} = 1/N = 0.015$, so that we are in the paramagnetic chaotic "phase". Dashed line is the linear regression with slope $-0.068$. The average has been taken over an ensemble of 10000 different samples. Initially all samples have all spins aligned along the $z$–axis : $S_i^z(0) = 1$, $i = 1, \ldots N$.

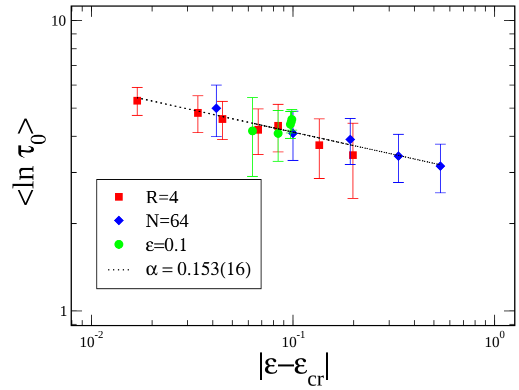


FIG. 7: Average first zero times *vs* $|\epsilon - \epsilon_{cr}|$, for different set of data as indicated in the legend. The dashed line is the best fit with exponent $\alpha = 0.153 \pm 0.016$. Initially we choose $S_i^z(0) = 1$, $i = 1, \ldots, N$. An ensemble of 1000 different configurations has been considered.

length $L$ and the number of spins $N$.

The numerical results are reported in Fig. 5 and agree with the following behavior,

$$\langle \ln\ \tau_0 \rangle \sim |\epsilon - \epsilon_{cr}|^{-\alpha}, \tag{13}$$

where $\alpha = 0.153 \pm 0.016$. This exponent is at variance with the exponents found[22], for which, typically $\alpha \sim N^{\sigma}$, where $\sigma$ depends on the interaction range. This indicates that the mechanism underlying this kind of transition, is different from the ergodicity breaking investigated in literature[3, 21, 22].

As one can see each point in Fig. 7 is characterized by an error bar, defined as the variance of the distribution of $ln\ \tau_0$. It is important to observe that, apparently, error bars do not increase on approaching the critical border. This is due to the fact that the variance of the probability distribution function depends on both the number of particles $N$ and the distance from criticality $|\epsilon - \epsilon_{cr}|$. Indeed, keeping fixed the distance $|\epsilon - \epsilon_{cr}|$ and varying the number of particles $N$ one finds a $1/\sqrt{N}$ dependence, as shown in Fig. 8a. Such dependence masks the real dependence of fluctuations, on approaching the critical border. Indeed, defining the renormalized variance $\sqrt{N}\Delta \ln \tau_0$, one gets a similar power law divergence at criticality for fluctuations, with an exponent very close to that found for the average time:

$$\Delta\langle \ln\ \tau_0 \rangle = \langle \ln^2\ \tau_0 \rangle - \langle \ln\ \tau_0 \rangle^2 \simeq |\epsilon - \epsilon_{cr}|^{-\beta}, \tag{14}$$

with $\beta = 0.187 \pm 0.039$.

This means that also fluctuations diverge at the critical point thus sharing another important feature with phase transitions. Unfortunately we have no theoretical arguments in order to predict the exponents $\alpha$ and $\beta$, and so, even if they are quite close one to each other we cannot infer any conclusion about the behavior of the relative variance on approaching the critical point.

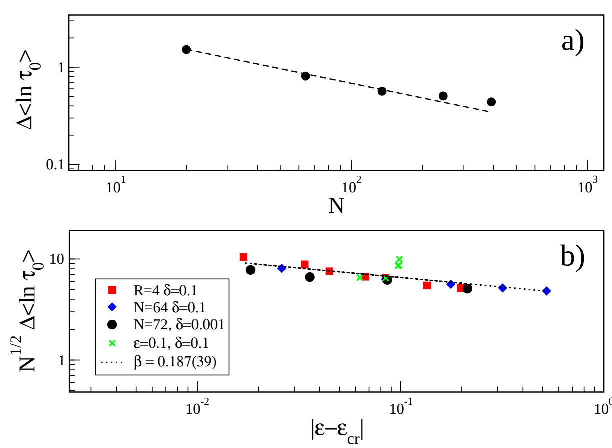


FIG. 8: a) Variance of the probability density function $\Delta \ln \tau_0$ for fixed $|\epsilon - \epsilon_{cr}| = 0.082$ as a function of the number of spins $N$. b) Renormalized variance $\sqrt{N}\Delta\langle \ln \tau_0 \rangle$ *vs* $|\epsilon - \epsilon_{cr}|$ for different series of data as indicated in the legend. Initially we choose $S_i^z(0) = 1$, $i = 1, \ldots, N$. An ensemble of 1000 different configurations has been considered for each point on the picture.

## V. CONCLUSIONS

In this paper the microcanonical dynamics of a system of random dipoles, interacting with a pure dipole-dipole interaction has been considered. Contrary to what happen for a system of random dipoles in the canonical ensemble, where a transition to a ferromagnetic phase occurs for high concentration, we have shown here, that a "phase" transition, correspondent to a transition from regular (ferromagnetic) to stochastic (paramagnetic) regime happens, in the microcanonical ensemble, for low concentration. Such transition is characterized as in similar systems investigated in literature, by a power law divergence of magnetic reversal times at the critical point. Supported by extensive numerical results we give an estimate of the critical point, but not of the power law exponents, which appear to be different from those reported in literature from similar models but characterizing the ergodicity breaking [21, 22].

In the future we intend to investigate dilute dipole systems in the canonical ensemble, that is letting the system be in contact with a thermal bath. Our analysis in the microcanonical ensemble indicated that the behavior of very dilute dipoles in a needle geometry is very similar to a 1–D arrays of dipoles. In the 1–D case dipole interaction induces a ferromagnetic ground state, and, due to its anisotropy, to a breaking of ergodicity [3]. As shown in previous papers[22], this ergodicity breaking threshold can induce very large magnetic reversal times thus producing ferromagnetic behavior in finite samples. Thus, even if one would expect that invariant tori will be destroyed under a suitable thermal perturbation, the question on the magnetic reversal times in presence of temperature and on the relevance of the ergodicity breaking is still open.

The ergodicity breaking discovered in Ref. [3] refers to the total magnetization as an order parameter. On the other side different order parameters can be defined in dipole systems, depending on the ground state configuration, for instance an anti-ferromagnetic order parameter or a spin glass order parameter. We intend to investigate the existence of an ergodicity breaking energy threshold with respect to different order parameters in a future paper.

In conclusion dipole-dipole interacting spin systems offer a realistic playground to analyze many properties of magnetic systems which challenge our comprehension.


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