EUROPHYSICS LETTERS 1 September 1996

Europhys. Lett., **35** (7), pp. 517-521 (1996)

Effect of noise for two interacting particles in a random potential

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(received 29 March 1996; accepted in final form 8 July 1996)

PACS. 71.55Jv – Disordered structures; amorphous and glassy solids.
PACS. 72.10Bg – General formulation of transport theory.
PACS. 05.45+b – Theory and models of chaotic systems.

Abstract. – We investigated the effect of noise on propagation of two interacting particles pairs in a quasi–one-dimensional random potential. It is shown that pair diffusion is strongly enhanced by short-range interaction comparing with the noninteracting case.

Recent investigations showed that two interacting particles (TIP) in a random potential can propagate coherently on a distance l_c which is much bigger than the one-particle localization length l_1 without interaction [1]-[6]. According to [1], [2] the TIP localization length is given by

$$l_c \sim l_1^2 M (U/V)^2/32 \,, \tag{1}$$

where M is the number of transverse channels in a quasi–one-dimensional wire, U is the strength of on-site interaction assumed to be comparable to or less than the bandwidth and V is the hopping matrix element between nearby sites. Here it is also assumed that the wave vector k_F corresponding to TIP energy is $k_F \sim 1/a = 1$, a being the lattice constant. While the exact verification of (1) is still under investigation, the existence of the enhancement of two-particle localization length have been clearly demonstrated in numerical simulations [3]-[6]. These simulations have been done for different models. Main results are for the 1-d Anderson model with TIP [3], [5] and for the model of two interacting kicked rotators [4]. This last model is very convenient for the investigation of wave packets spreading in time due to the existing effective numerical methods. For this reason, in our numerical studies, we used the last model.

The problem we want to address in this letter is the influence of noise on TIP localization. For one particle the effect of noise has been analyzed during the last few years and the physics

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of this phenomenon is well understood [7]-[10]. Generally, the main effect of noise is the destruction of interference after a coherence time t_c, after which the particle makes a jump on a distance l_1 that leads to a diffusion rate $D_1 \sim l_1^2/t_c$ along the chain. For high-frequency noise with amplitude ϵ this time is $t_c \propto 1/\epsilon^2$ [7]. In the case of low-frequency (ω) noise $t_c \propto 1/(\epsilon\omega)^{2/3}$ [10]. Here we only analyzed the case of high-frequency noise on the TIP problem.

We studied the model of two interacting kicked rotators in the presence of noise. The evolution operator is given by

$$\hat{S} = \exp\left[-i[H_0(\hat{n}_1) + H_0(\hat{n}_2) + U\delta_{n_1,n_2}]\right] \times \exp\left[-i[V(\theta_1, t) + V(\theta_2, t)]\right], \tag{2}$$

where $\hat{n}_{1,2} = -i\partial/\partial\theta_{1,2}$, $H_0(\hat{n}) = T\hat{n}^2/2$, $V(\theta, t) = [k + \epsilon f(t)]\cos\theta$, and $f(t)$ is a random function of t homogeneously distributed in the interval $[-1, 1]$.

For $U = 0$ and $\epsilon = 0$ we have two decoupled kicked rotators and in the chaos domain $kT > 1$ the localization length is $l_1 \approx k^2/2$ [11]. In the presence of noise $\epsilon > 0$ the decoherence time is $t_c \sim 1/\epsilon^2$. If this time is less than the localization time $t_1^* \approx l_1$, localization effects are not important and diffusion goes with the usual classical rate $D \approx k^2/2$. On the other hand, when $t_1^* < t_c$ the diffusion rate is $D_1 \sim \epsilon^2 l_1^2$ [7].

In the presence of interaction ($U \neq 0$), but without noise ($\epsilon = 0$), a TIP pair of size l_1 propagates on a distance $l_c \gg l_1$ and is localized after a time $t_2^* \sim l_c l_1$ [12]. Over a time interval $t_1^* < t < t_2^*$ the pair diffuses with a diffusion rate $D_p \sim U^2 D$. The noise leads to a destruction of localization after a decoherence time $t_c \sim 1/\epsilon^2$ as in the one-particle case. Indeed, without interaction this time is independent of localization length (l_1) and lattice dimension. Due to that we assume, in agreement with our numerical results, that t_c is also independent of interaction. After the time t_c the pair makes a jump of size l_c and therefore the noise-induced pair diffusion rate can be estimated as $D_+ \sim l_c^2/t_c$ that gives for $t_c > t_2^*$

$$D_+ \sim \epsilon^2 l_c^2 \sim D_1 (l_c/l_1)^2. \tag{3}$$

This means that for $l_c/l_1 \gg 1$ the noise-induced TIP diffusion rate is strongly enhanced with respect to the noninteracting one (D_1). In the case of relatively strong noise, $t_c < t_2^*$ and D_+ becomes comparable with the pair diffusion rate D_p on a time scale $t_1^* < t < t_2^*$.

The estimate (3) gives the diffusion rate of the center of mass of the TIP pair. However, noise also leads to a separation of two particles. This separation goes in a diffusive way with a diffusion rate D_1 which is independent of the interaction U. Due to this, at asymptotically large times the pair propagation will be subdiffusive. At this time the spreading of the center of mass Δn_+^2 can be estimated in the following way: $\Delta n_+^2 \sim \nu D_+ t$, where ν is the probability of collision between two particles, $\nu \sim l_1/\Delta n_-$. Here Δn_- is the effective pair size at the time t which in turn can be estimated as $\Delta n_-^2 \sim D_1 t$. Therefore, we have $\Delta n_+^2 \sim (l_c/l_1)^2 l_1 \sqrt{D_1 t}$. Comparing this result with the diffusive growth given by (3) $\Delta n_+^2 \sim D_+ t$, we determine the time scale $t_+ \sim 1/\epsilon^2$ during which the pair propagates diffusively with the rate D_+. This time is parametrically comparable with t_c and according to our estimates can be only numerically larger than t_c. Nevertheless, even with the separation of particles due to diffusion produced by noise the effect of interaction is quite important. Indeed, the time t_{int} after which interaction becomes nonsignificant is quite large. Namely, it can be found from the condition that the noninteracting diffusive spreading $\Delta n_+^2 \sim D_1 t_{int}$ for $t > t_{int}$ is comparable with the interacting case where $\Delta n_+^2 \sim (l_c/l_1)^2 l_1 \sqrt{D_1 t_{int}}$ that gives a very large time scale $t_{int} \sim (l_c/l_1)^4/\epsilon^2 \gg t_c$.

The numerical check of all these different time scales is quite difficult due to the strong growth of the required basis. Therefore, we numerically studied only the regime of short times, when $\Delta n_- \sim l_1$ and the diffusive spreading in n_+ is still given by $\Delta n_+^2 \sim D_+ t$. Examples of these diffusive behaviours are presented in fig. 1 and fig. 2, where the growth

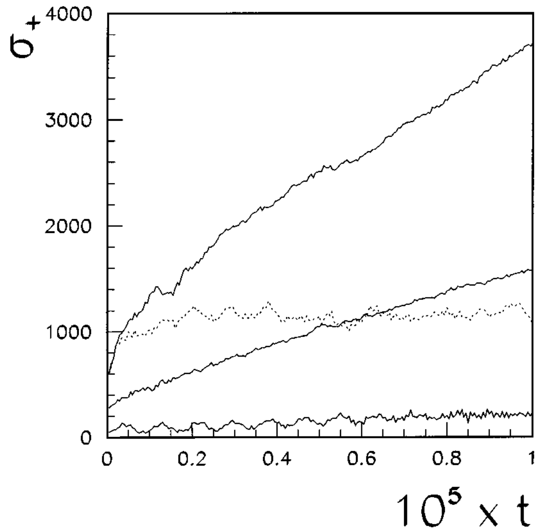

Fig. 1. – Dependence of σ_+ on time for different U values: $U = 0, 1, 2$ correspond to lower, middle and upper full curves, $\epsilon = 0.02, k = 4, kT = 5$. The dotted line is for $\epsilon = 0, U = 2, k = 4, kT = 5$. Initially particles are at $n_1 = n_2 = 0$. The basis is $-250 < n_{1,2} < 250$.

of $\sigma_+ = (|n_1| + |n_2|)^2/4$ and $\sigma_- = (|n_1| - |n_2|)^2$ is shown as a function of time. These results clearly show that the diffusion rate $D_+ = \sigma_+/t$ is strongly enhanced with the interaction switched on (approximately 20 times from $U = 0$ to $U = 2$). At the same time the diffusion rate in n_- ($D_1 = \sigma_-/t$) remains practically the same (see fig. 2). For the sake of comparison we also present in fig. 1 and 2 TIP localization in the absence of noise, when $\sigma_\pm$ are oscillating in time near their asymptotic values.

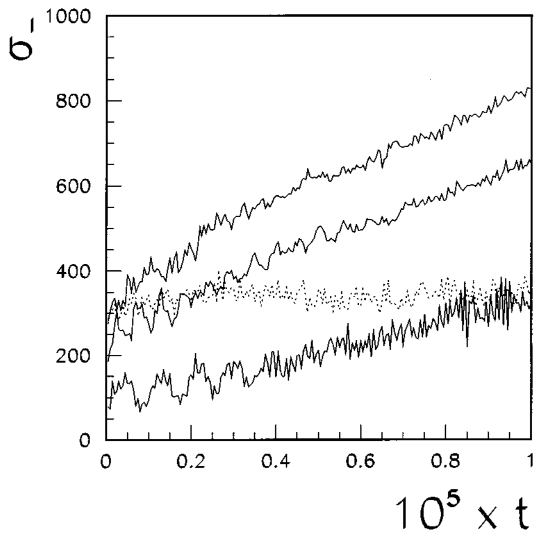

Fig. 2. – The same as fig. 1 but for σ_-.

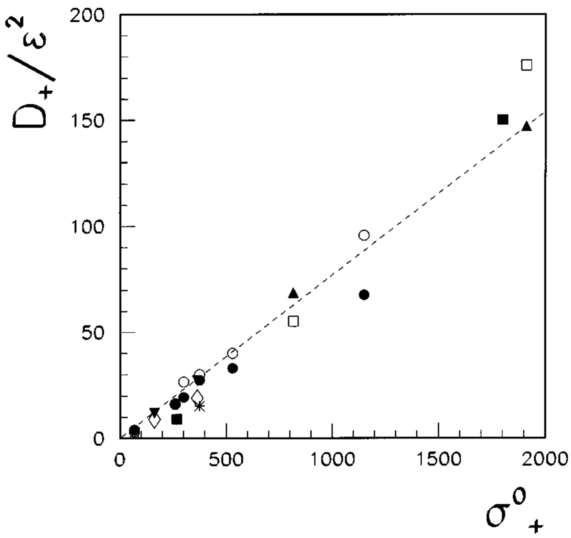

Fig. 3. – Dependence of the TIP pair diffusion rate on σ^0_+. Full circles are for $k = 4, \epsilon = 0.01$, $0 \le U \le 2$, open circles for $k = 4, \epsilon = 0.02, 0 \le U \le 2$, full squares for $k = 5.7, \epsilon = 0.01, U = 0, 1$, open squares for $k = 4.8, \epsilon = 0.02, U = 1, 2$, up-triangles for $k = 4.8, \epsilon = 0.01, U = 1, 2$, down-triangles for $k = 3.3, \epsilon = 0.01, U = 1, 2$, diamonds for $k = 3.3, \epsilon = 0.03, U = 1, 2$, asterisks for $k = 4, \epsilon = 0.05, U = 0, 1$. In all cases the chaos parameter has been fixed, $kT = 5$. The dashed line shows the average dependence $D_+ = 13\epsilon^2\sigma^0_+$.

To check the relation (3), we determined the diffusion rate D_+ and checked its dependence on parameters. According to (3) $D_+ \sim \epsilon^2\sigma^0_+$, where $\sigma^0_+ \sim l_c^2$ is the asymptotic value of σ_+ in the absence of noise. The dependence of D_+/ϵ^2 on σ^0_+ is shown in fig. 3 for different ϵ, U and k values. The average behaviour is given by the approximate relation $D_+ = 13\epsilon^2\sigma^0_+$ and it is in agreement with the theoretical prediction (3).

In real systems, noise can appear as the result of electron interaction with phonons at finite temperature. Our results indicate that the noise produced by phonons can lead to a strong enhancement of diffusion (conductance) of electrons in a random potential. Of course in the analysis of a physical model the case of finite particles (or quasi-particles) density should be considered. In this case the probability to find two particles within a distance l_1 from each other is of the order of $\mathcal{W} \sim l_1\rho$, where ρ is the linear (per unit length) density of particles. This leads to a decrease of the effective diffusion rate which in this case can be estimated as $D_{\text{eff}} \sim \mathcal{W}D_+ \sim D_1 l_c^2\rho/l_1$. Even for small density D_{eff} can be larger than D_1 and we expect that the effect of interaction-enhanced diffusion is physically relevant. However, future investigations on the finite-particles-density case should be done.

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We acknowledge the University of Como for hospitality during the final stage of this work.

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