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Particle propagation in a random and quasi-periodic potential

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Abstract

We numerically investigate the Anderson transition in an effective dimension d ($3 \leq d \leq 11$) for one particle propagation in a model random and quasi-periodic potential. The found critical exponents are different from the standard scaling picture. We discuss possible reasons for this difference.

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The Anderson transition has been intensively investigated during last years. According to the scaling theory for spinless particles all states are localized for dimension $d \leq 2$, while a transition from localization to diffusive propagation occurs for $d > 2$ when a certain hopping parameter k crosses a critical value k_{cr} (see reviews [1,2]). The one parameter scaling theory predicts that in the localized phase ($k < k_{cr}$) the typical localization length of wavefunctions diverges at the critical point as $l \sim |k - k_{cr}|^{-\nu}$. Above the critical point the dynamics is diffusive and the DC conductivity σ which is proportional to the diffusion rate D is assumed to approach zero at the critical point as $\sigma \propto D \sim |k - k_{cr}|^{s}$. Scaling arguments give the following relation between the exponents: $s = (d-2)\nu$. For dimension $d = 2 + \epsilon$ the ϵ expansion theory predicts $s = \nu = 1 + O(\epsilon^4)$ [3]. In higher dimensions the problem was studied by many authors [4–6]. According to the results presented there $\nu = 1/2$ for $d \to \infty$ while for the delocalized phase $s \approx d/2$ [6] or σ decreases exponentially near the transition [4].

Numerical investigations of the exponents has been restricted to $d = 3$ where it was found $s = \nu = 1.5 \pm 0.1$ ([2] and reference therein) in agreement with the scaling relation between ν and s. However, the applied numerical methods were quite heavy and did not allow to obtain a better accuracy in the determination of the exponents or to increase the dimension. For $2 < d \leq 3$ and $d = 4$ there are only recent results for ν [7] obtained by transfer matrix technique. While the results there seem to be in agreement with the scaling theory the system size was so small that the question if the thermodynamic limit had been reached remains open.

An effective way to increase the number of dimensions was proposed in [8] and applied for investigation of transition in $d = 3$ [9] where it was also found $s \approx 1.25$ and $\nu \approx 1.5$. The method consists in the investigation of the well-known model of quantum chaos

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