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Two Interacting Particles in an Effective 2-3–d Random Potential

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Abstract. — We study the effect of coherent propagation of two interacting particles in an effective 2-3-d disordered potential. Our numerical data demonstrate that in dimension $d > 2$, interaction can lead to two–particles delocalization below one–particle delocalization border. We also find that the distance between the two delocalized particles (pair size) grows logarithmically with time. As a result pair propagation is subdiffusive.

1. Introduction

The question of interacting particles in a random potential has recently got a great deal of attention (see for example [1]). Indeed this problem is important for the understanding of conduction of electrons in metals and disordered systems. It is also very interesting from the theoretical view point since it allows to understand the effects of interaction on Anderson localization. It is a common belief that in one-dimensional (1d) systems near the ground state, a repulsive interaction between particles leads to a stronger localization if compared with non interacting case [2]. Even if more complicate, the 2–dimensional (2d) problem is assumed to be localized, while in the 3–dimensional (3d) case delocalization can take place in the presence of interaction [1]. However this problem is rather difficult for analytical, experimental and numerical investigations and therefore it is quite far from its final resolution.

The complicated nature of the above problem can be illustrated by the example of only two interacting particles (TIP) in a random potential. Indeed in this case, contrary to the common lore, even repulsive particles can create an effective pair which is able to propagate on a distance l_c much larger than its own size of the order of one particle localization length l_1 [3]. From one side interference effects for non–interacting particles force the particles to stay together at a distance $\sim l_1$ even in the repulsive case. But the relative motion between the two particles leads to the destruction of such interference and allows their coherent propagation on a distance $l_c \gg l_1$. More explicitly, according to [3], in the quasi 1d case with M transverse

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channels one has

$$\frac{l_c}{l_1} \sim l_1 M \frac{U^2}{32V^2} \tag{1}$$

where U is the strength of on site interaction and V is the one particle hopping matrix element. Here the inter-site distance is $a = 1$ and the wave vector $k_F \sim 1$. Since $l_1 \propto M$, then $l_c \propto M^3$. In 2d localization is preserved but localization length is exponentially large : $\ln(l_c/l_1) \sim l_1^2$ [6]. (here and everywhere l_1 represents the one–particle localization length in any dimension). The sharp increase of l_c with the number of transverse channels M leads to a straightforward possibility of delocalization for a pair of particles in 3d while one particle remains localized [4–6]. In this sense the 3d case is much more interesting due to the possibility of delocalization and we qualitatively discuss it below.

One of the interesting features of pair delocalization in 3d is that it is not due to a simple shift of the mobility edge produced by the interaction (such possibility is indeed not so interesting). In fact, it is possible to consider a system in which *all* one-particle eigenstates are localized for *all* energies. This can be for example the 3d Lloyd model with diagonal disorder $E_{n_1,n_2,n_3} = \tan \phi_{n_1,n_2,n_3}$ and hopping V on a cubic lattice, where ϕ_{n_1,n_2,n_3} are random phases homogeneously distributed in the interval $[0, \pi]$. In this model *all* one-particle eigenstates are localized for $V < V_c \sim 0.2$. However, two repulsive particles with on-site interaction U can create a coupled state which is delocalized and propagates through the lattice. Indeed, a pair "feels" only smoothed potential [3] that corresponds to an effective renormalization of the hopping matrix element V_{eff} which is strongly enhanced due to interaction and becomes larger than V_c. Of course, the enhancement takes place only for sufficiently large one-particle localization length $l_1 \gg 1$. Therefore, the hopping $V < V_c$ should be not far from V_c although there is, *a priori*, no requirement for V to be very close (parametrically) to V_c.

The two particles delocalization due to interaction takes place only for states in which particles are on a distance $R < l_1$ from each other while for $R \gg l_1$ eigenstates are localized. Such kind of situation is quite unusual since it means that the absolutely continuous spectrum of the Schrödinger operator, corresponding to the delocalized pair, is *embedded* into the pure point spectrum of localized almost noninteracting particles states. For $R \gg l_1$ the interaction between the two particles is exponentially small and this implies a very small coupling between the states corresponding to these two kinds of spectra. However, due to the quasi degeneracy of levels, even a small coupling can lead to important modifications of the above picture as it was discussed in [6]. Therefore, a direct numerical investigation of interaction-assisted delocalization in 3d is highly desirable. While the recent theoretical arguments and numerical simulations in quasi–1d case [3] - [9] definitely demonstrate the existence of enhancement for l_c no numerical simulations have been done in 3d case. Indeed, in 3d basis grows as N^6, where N is the number of 1d unperturbed one-particle states, and that leads to heavy numerical problems.

A similar type of interaction-assisted delocalization can be also realized in the kicked rotator model (KRM) [10] in 3d. In this case the unitary evolution operator takes the place of Schrödinger operator and eigenenergies are replaced by quasi-energies. The advantage of such models is due to the independence of localization length on quasi-energy so that all one-particle states in 3d are localized for $V < V_c$ and delocalized for $V > V_c$. Even if very efficient, numerical simulations for KRM in 3d become very difficult; for two particles situation becomes even worse due to N^6 basis growth.

One of the ways to overcome these numerical difficulties is the following. For 1d KRM the number of dimensions can be effectively modelled by introducing a frequency modulation of the perturbation parameter [11, 12]. The case with ν incommensurate frequencies in the kick modulation corresponds to an effective solid state model with dimension $d = \nu + 1$. For $\nu = 2$,

the effective dimension $d = 3$ and Anderson transition can be efficiently investigated [12]. A similar approach can be done for two interacting particles and it allows to gain a factor N^4 in numerical simulations.

In this paper we investigate the model of two interacting kicked rotators (KR) studied in [3], [6] with frequency modulation and $\nu = 2, 3$. The quantum dynamics is described by the evolution operator :

$$\begin{aligned} \hat{S}_2 &= \exp\{-i[H_0(\hat{n}) + H_0(\hat{n}') + U\delta_{n,n'}]\} \\ &\times \exp\{-i[V(\theta,t) + V(\theta',t)]\} \end{aligned} \tag{2}$$

with $\hat{n}^{(')} = -i\partial/\partial\theta^{(')}$. Here $H_0(n)$ is a random function of n in the interval $[0, 2\pi]$ and it describes the unperturbed spectrum of rotational phases. The perturbation V gives the coupling between the unperturbed levels and has the form $V(\theta,t) = k(1+\epsilon\cos\theta_1\cos\theta_2\cos\theta_3)\cos\theta$ with $\theta_{1,2,3} = \omega_{1,2,3}\ t$. In the case of two modulational frequencies ($\nu = 2, \omega_3 = 0$), as in [12], we choose frequencies $\omega_{1,2}$ to be incommensurate with each other and with the frequency 2π of the kicks. Following [12] we take $\omega_1 = 2\pi\lambda^{-1}$, $\omega_2 = 2\pi\lambda^{-2}$ with $\lambda = 1.3247...$ the real root of the cubic equation $x^3 - x - 1 = 0$. For $\nu = 3$ we used the same $\omega_{1,2}$ and $\omega_3 = 2\pi/\sqrt{2}$. We also studied another case of functional dependence of $V(\theta)$ analogous to [12] and corresponding to the Lloyd model. All computations have been done for symmetric configurations. According to the theoretical arguments [3] and numerical simulations [9] the antisymmetric configurations corresponding to fermions with nearby site interaction should show a similar type of behaviour.

The paper is constructed as follows. In Section 2 we discuss the model and present the main results for $\nu = 2$. The case of $\nu = 3$ is discussed in Section 3. The kicked rotator model corresponding to the 3d Lloyd model is studied in Section 4. Conclusions and discussions of results are presented in Section 5.

2. The KRM Model with Two Frequencies

Before to discuss the effects of interaction let us first discuss the noninteracting case $U = 0$. Here the evolution operator can be presented as a product of two operators S_1 describing the independent propagation of each particle:

$$\hat{S}_1 = \exp(-iH_0(\hat{n}))\exp(-iV(\theta,t)) \tag{3}$$

Since V depends on time in a quasiperiodic way with $V(\theta,t) = V(\theta,\theta_1,\theta_2)$ and $\theta_{1,2} = \omega_{1,2}t$, one can go to the extended phase space [11], [12] with effective dimension $d = 3$. In this space the operator is independent on time and has the form

$$\hat{S}_1 = \exp(-iH_1(\hat{n},\hat{n}_1,\hat{n}_2))\exp(-iV(\theta,\theta_1,\theta_2)) \tag{4}$$

with $H_1(n,n_1,n_2) = H_0(n) + \omega_1 n_1 + \omega_2 n_2$. Due to linearity in $n_{1,2}$ the transformation from (3) to (4) is exact. However, the numerical simulations of (3) are N^2 times more effective than for (4).

The system (4) corresponds to an effective 3d model. Numerical simulations in [12] showed that the variation of coupling amplitude V gives the transition from localized to diffusive regime as in usual Anderson transition in $3d$. In [12] the form of the kick V had been chosen as

$$V(\theta,\theta_1,\theta_2) = -2\tan^{-1}[2k(\cos\theta + \cos\theta_1 + \cos\theta_2) - E] \tag{5}$$

In this case after a mapping similar to the one used in [13] the equation for eigenfunction with a quasi-energy μ can be presented in a usual solid-state form:

$$T_{\mathbf{n}}u_{\mathbf{n}} + k\sum_{\mathbf{r}} u_{\mathbf{n}-\mathbf{r}} = Eu_{\mathbf{n}} \tag{6}$$

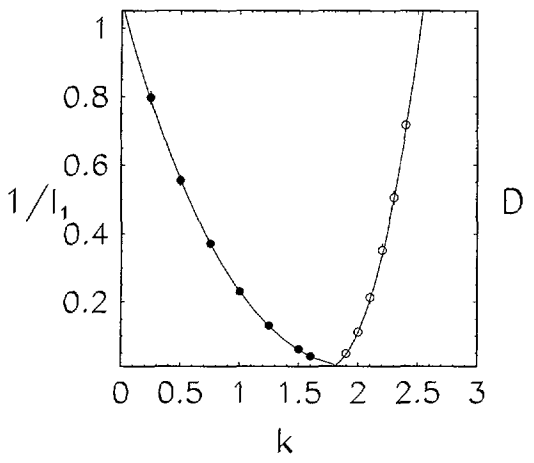

Fig. 1. — One-particle Anderson transition in the model (3) with V from (7) $\nu = 2$, $\epsilon = 0.75$ as a function of hopping k. Critical point $k_{cr} \approx 1.8$. One particle localization lengths (l_1) and diffusion coefficients (D) are evaluated from the fitting of probability distribution. Error bars indicate the standard deviation obtained from an ensemble of 100 (localized) and 10 (diffusive) different random realizations. Lines are drawn to fit an eye.

where the sum is taken only over nearby sites and $T_{\mathbf{n}} = \tan((H_1(n, n_1, n_2) - \mu)/2)$, $\mathbf{n} = (n, n_1, n_2)$. For random phases under tangent the diagonal disorder is distributed in Lorentzian way and the model becomes equivalent to the 3d Lloyd model. While we also investigated the kick form (5) (see Section 4) our main results have been obtained for

$$V(\theta, \theta_1, \theta_2) = k \cos\theta(1 + \epsilon \cos\theta_1 \cos\theta_2) \tag{7}$$

in the case of two frequencies $\nu = 2$. According to [14] in this case the equation for eigenfunctions can be also reduced to an effective solid-state Hamiltonian which, however, has a bit more complicated form than (6). We choose (7) since it was numerically more efficient than (5). To decrease the number of parameters we always kept $\epsilon = 0.75$.

The one-particle transition as a function of coupling (hopping) parameter k in (7) is presented in Figure 1. Similar to [12] the localization length l_1 is determined from the stationary probability distribution over unperturbed levels $|\psi_n|^2 \sim \exp(-2|n|/l_1)$ while the diffusion rate is extracted from the Gaussian form of the probability distribution $\ln W_n \sim -n^2/2Dt$ with $D =< n^2 > /t$. According to Figure 1 the transition takes place at the critical hopping value $k_{cr} \approx 1.8$. Below k_{cr} all quasi-energy states are localized. The independence of transition point from quasi-energy is one of useful properties of KR models.

Our main aim was the investigation of TIP effects well below the transition point k_{cr}. As in [6] we characterized the dynamics (2) by the second moments along the diagonal line $n = n'$: $\sigma_+(t) = \langle(|n| + |n'|)^2\rangle_t/4$ and across it $\sigma_-(t) = \langle(|n| - |n'|)^2\rangle_t$. We also computed the total probability distribution along and across this diagonal [6]: $P_\pm(n_\pm)$ with $n_\pm = |n \pm n'|/2^{1/2}$ The typical case is presented in Figures 2 and 3. These pictures definitely show the appearance of pair propagation even if the interaction is neither attractive nor repulsive. Indeed, the pair size is much less than the distance on which two particles are propagating together. If we fit the probability distribution in Figure 3 as $P_\pm \sim \exp(-2n_\pm/l^\pm)$ then we can see that the ratio $l^+/l^- \sim 25$ ($l_c \approx l^+ \approx 95$) is quite large. It is interesting to note that $P_+(n_+)$ at different moments of time is closer to an exponential ($\ln P_+ \sim n_+$) than to a Gaussian

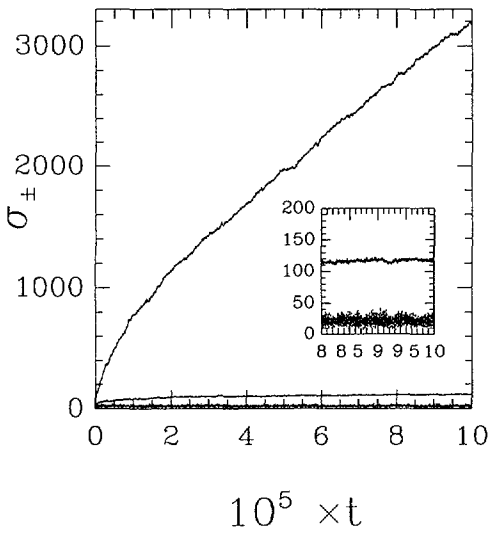

Fig. 2. — Dependence of second moments on time in 2-particles model (2) with V from (7) and $\nu = 2$, $k = 0.9$, $\epsilon = 0.75$,; upper curve is $\sigma_+(U = 2)$, middle is σ_- $(U = 2)$, lower is σ_+ $(U = 0)$. At $t = 0$ both particles are at $n = n' = 0$, basis is $-250 \leq n, n' \leq 250$. Insert shows in larger scale the two lower curves in the interval $8 \times 10^5 < t < 10^6$.

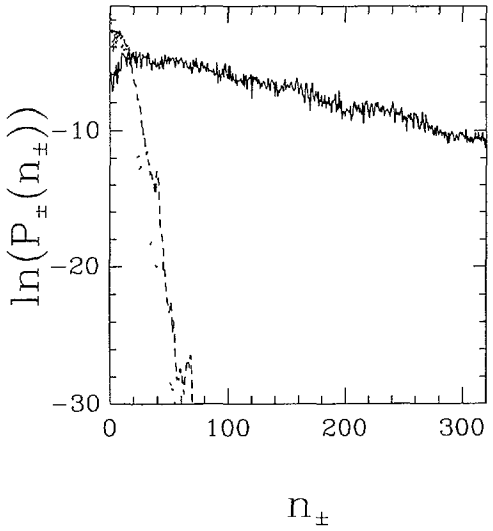

Fig. 3. — Probability distribution at $t = 10^6$ as a function of $n_{\pm} = 2^{-1/2}(n \pm n')$ for the case of Figure 2 with $U = 2$: $P_+(n_+)$ (full line); $P_-(n_-)$ (dashed); dotted line is the distribution $P_+(n_+)$ for $U = 0$.

$(\ln P_+ \sim {n_+}^2)$. The spreading along the lattice leads only to growth of l^+ with time but the shape of distribution does not corresponds to a diffusive process.

Another interesting feature of Figure 2 is the slow decrease of the rate of σ_+ growth and the slow growth of σ_-. To check if the growth of σ_+ is completely suppressed with time we analysed

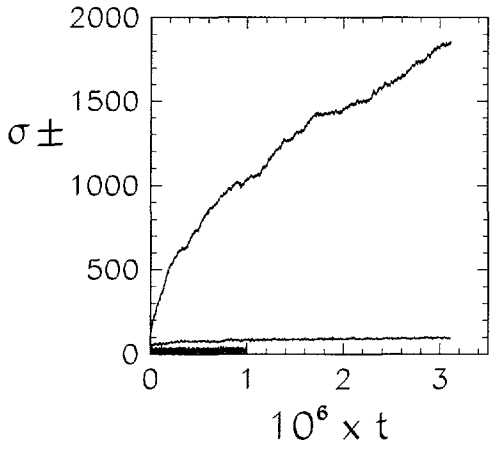

Fig. 4. — Same as Figure 2 but for $U = 1$.

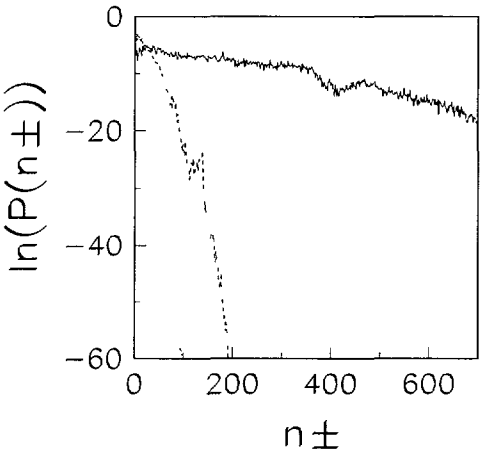

Fig. 5. — Same as Figure 3 but for $U = 1$.

its dependence on t for different values of interaction U (Fig. 4, probability distribution is shown in Fig. 5). For $U \leq 0.5$ the growth of σ_+ is completely suppressed while for $U = 1$ the complete suppression is a bit less evident. To understand in a better way the case $U = 1$ we can look on the dependence of the number of effectively excited levels ΔN on time. To estimate ΔN we should rewrite (2) in the extended basis where the evolution operator has the form:

$$\hat{S}_2 = \exp(-i[H_0(\hat{n}) + H_0(\hat{n}') + \omega_1\hat{n}_1 + \omega_2\hat{n}_2 + U\delta_{n,n'}]) \exp(-i[V(\theta,\theta_1,\theta_2) + V(\theta',\theta_1,\theta_2)]) \quad (8)$$

Since $\Delta n_{1,2} \approx \Delta n^{(\prime)} \approx \Delta n_+$ the number of excited levels can be estimated as $\Delta N \approx \Delta n_+ \Delta n_- \Delta n_1 \Delta n_2 \approx {\sigma_+}^{3/2}{\sigma_-}^{1/2}$. Following the standard estimate based on the uncertainty relation [14] a delocalization can take place only if ΔN grows faster than the first power of t. As our numerical data show (see Fig. 6) the ratio $W = \Delta N/t$ remains approximately constant or is even slightly decreasing in time. This indicates that our case is similar to localization in $2d$ where this ratio also remains constant for very long time. The reason why below $k_{\rm cr}$ the

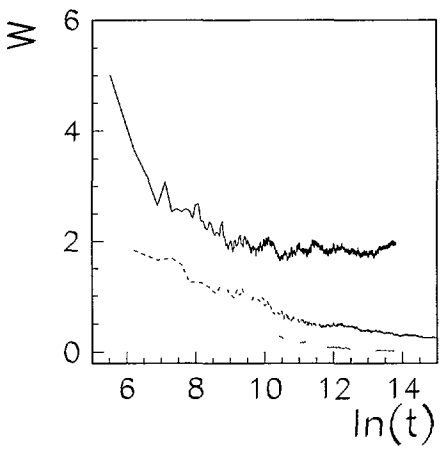

Fig. 6. — Dependence of $W = \Delta N/t = {\sigma_+}^{3/2}{\sigma_-}^{1/2}/t$ on time for $\nu = 2$, $k = 0.9$, $\epsilon = 0.75$ and $U = 2$ (full line), $U = 1$ (dashed line), $U = 0.5$ (dotted line).

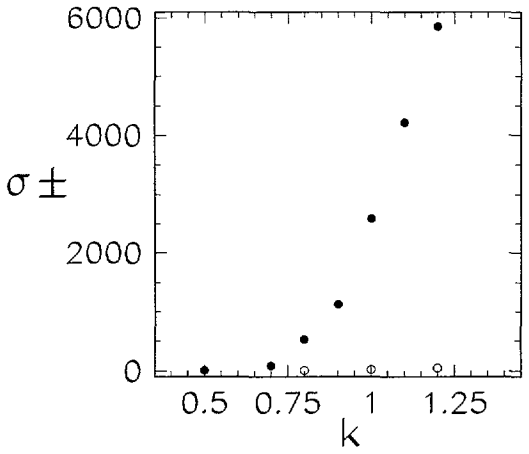

Fig. 7. — Dependence of σ_+ on k for $\nu = 2$ at $t = 2 \times 10^5$, $U = 2$ (full circles), $U = 0$ (open circles).

situation is similar to $2d$ can be understand in the following way. According to (8) the total dimension is 4 and we have there 2 particles. Therefore, we can argue that the dimension per particle is 2 and that below the 3d delocalization border k_{cr} our system effectively represents two particles in an effective dimension $d_{eff} = 2$. However, two particles in 2d are always localized but the localization length can be exponentially large. The dependence of σ_+ at fixed moment of time on k is presented in Figure 7 and indeed, it demonstrates a sharp increase of σ_+ with l_1 approaching k_{cr}. Therefore, we conclude that for $k < k_{cr}$ our model effectively represents TIP in 2d. Below k_{cr} the pair created by interaction remains localized but the localization length l_c grows exponentially with l_1. To see effects of interaction in $d_{eff} > 2$ we should study the system with three modulational frequencies $\nu = 3$. But before to analyse the case $\nu = 3$ we would like to discuss the behaviour of σ_-.

Indeed, Figures 2 and 4 clearly demonstrate a slow growth of σ_- with time which means

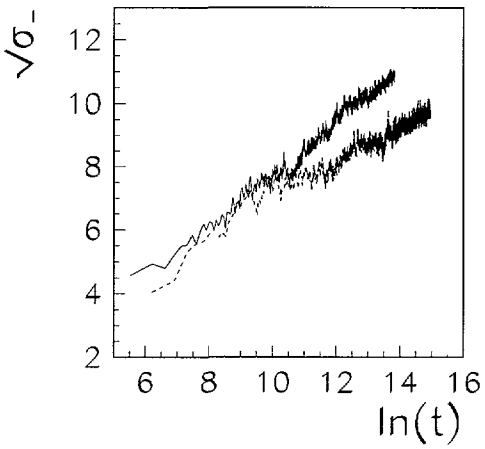

Fig. 8. — Dependence of $\sigma_-{}^{1/2}$ on time $\ln t$ for cases of Figure 2 ($U = 2, k = 0.9$) (full upper line) and Figure 4 ($U = 1, k = 0.9$) (dashed lower line).

the increase of the size of the pair κ. The results presented in Figure 8 show that $\kappa \approx \sigma_-{}^{1/2}$ grows logarithmically with time as $\kappa \approx C_L \ln t$ where C_L is some time independent factor being $C_L \approx 0.8(U = 2)$ and $C_L \approx 0.6(U = 1)$. Of course, this logarithmic growth should terminate after the complete localization in σ_+ but this time scale t_c is very large and for $t < t_c$ we have clear logarithmic growth of κ. As discussed in [6] we attribute this growth to the fact that propagating in a random potential the pair is affected by some effective noise which leads to a slow separation of two particles. Indeed, the matrix elements of interaction U_- decay exponentially fast with the growth of the pair size $n_- = \kappa$ according to a rough estimate $U_- \sim U_s \exp(-|n_-|/l_1)$ with $U_s \sim U/{l_1}^{3/2}$. These small but finite matrix elements lead to the growth of the pair size κ with slow diffusion rate $D_- \propto {U_s}^2 \exp(-2|n_-|/l_1)$. According to the relation $\kappa^2/t \approx D_-$ the pair size grows as $\kappa \sim l_1 \ln t/2$ [6] which is in agreement with data of Figure 7. More detailed numerical simulations are required to verify the dependence $C_L \sim l_1$.

Let us now discuss the physical interpretation of the model (8) . Firstly we would like to mention that in 1–d case, instead of looking on the problem of TIP in the same random potential, one can analyze the model where each particle is moving in its own independent random potential (two parallel random chains). Without interaction particles are independently localized on a length l_1, while in the case with interaction $U\delta_{n,n'}$ between the two chains, the particle pair can propagate on a larger distance $l_c \sim l_1^2$. Since the potentials are different in the two chains, there are no correlations in the matrix elements and all assumptions used for TIP in one random chain are even better justified. From this point of view the model (8) corresponds to the case in which each particle is moving in its own 2–d plane (two parallel planes with different realization of disorder). The interaction between the two particles in the planes will lead to the enhancement of localization length.

One can argue that (8) still contains some non-interactive transitions between the two planes, but we think that below one-particle delocalization border these transitions are not of principal importance and do not lead to significant changes. In the same spirit one can analyze the case with a larger number of frequencies, corresponding to higher dimensions.

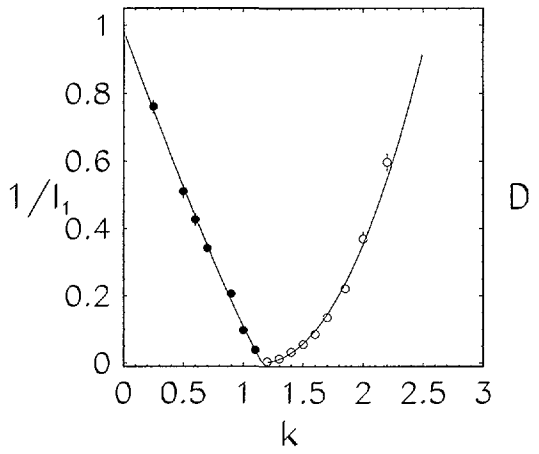

Fig. 9. — One-particle Anderson transition in the model (3) with V from (10) $\nu = 3$, $\epsilon = 0.9$ as a function of hopping k. Critical point $k_{\rm cr} \approx 1.15$. One particle localization lengths (l_1) and diffusion coefficient (D) are evaluated from the fitting of probability distribution. Error bars indicate the standard deviation for ensemble of 100 (localized) and 10 (diffusive) different random realizations. Lines are drawn to fit an eye.

3. The KRM Model with Three Frequencies

According to the above discussion the suppression of diffusive growth of σ_+ can be explained by two factors. The first one is that the effective dimension is $d_{\rm eff} = 2$ and localization always takes place in 2d. Another reason is the slow logarithmic growth of pair size. To separate these two effects we investigated the dynamics of TIP in the KRM with three modulational frequencies $\nu = 3$. In the extended phase space the evolution operator has the form

$$\begin{aligned}\hat{S}_2 \; = &\exp(-i[H_0(\hat{n}) + H_0(\hat{n}') + \omega_1\hat{n}_1 + \omega_2\hat{n}_2 + \omega_3\hat{n}_3 + U\delta_{n,n'}]) \\ &\exp(-i[V(\theta,\theta_1,\theta_2,\theta_3) + V(\theta',\theta_1,\theta_2,\theta_3)])\end{aligned} \tag{9}$$

with

$$V(\theta,\theta_1,\theta_2) = k\cos\theta(1 + \epsilon\cos\theta_1\cos\theta_2\cos\theta_3) \tag{10}$$

For $U = 0$ we have one particle in 4d and transition to delocalization takes place above a critical value of perturbation parameter $k_{\rm cr}$. According to our numerical data $k_{\rm cr} \approx 1.15$ for $\epsilon = 0.9$ (Fig.9). Below $k_{\rm cr}$ all eigenstates are exponentially localized. For $U \neq 0$ the total dimension in (9) is 5 and since we have 2 particles the effective dimension per particle is $d_{\rm eff} = 5/2$. Since $d_{\rm eff} > 2$ the first argument given above becomes not relevant and we expect TIP delocalization below $k_{\rm cr}$. Let us to note that above $k_{\rm cr}$ the above TIP problem becomes not interesting since even without interaction the particles spread along the lattice and the interaction between them does not affect significantly their dynamics.

The numerical simulations of TIP for (9)-(10) in one-particle localized phase $k < k_{\rm cr}$ demonstrate strong enhancement of two particles propagation. A typical case is presented in Figure 10 and 11. According to these data the growth of σ_+ is unlimited and TIP delocalization takes place below $k_{\rm cr}$. The analysis of $\sigma_-(t)$ shows that pair size $\kappa \approx \sigma_-^{1/2}$ grows logarithmically with time similar to the case with $\nu = 2$. We think that this slow growth of κ is responsible for a slow decrease of the pair diffusion rate $D_+ = \sigma_+/t$ with time. Due to the

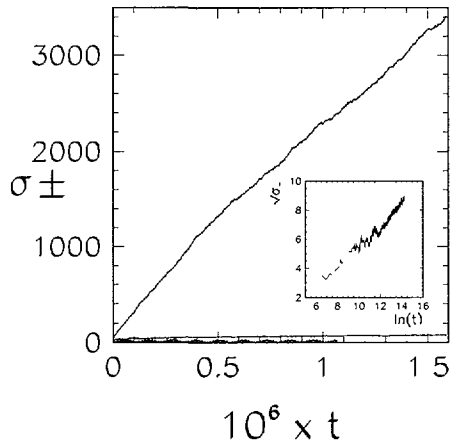

Fig. 10. — Dependence of second moments on time in model (2) with V from (10) and $k = 0.7$, $\epsilon = 0.9$; upper curve is σ_+ ($U = 2$), middle is σ_- ($U = 2$), lower is σ_+ ($U = 0$). At $t = 0$ both particles are at $n = n' = 0$, basis is $-250 \leq n, n' \leq 250$, $\nu = 3$. Inset shows the dependence of ${\sigma_-}^{1/2}$ on $\ln(t)$.

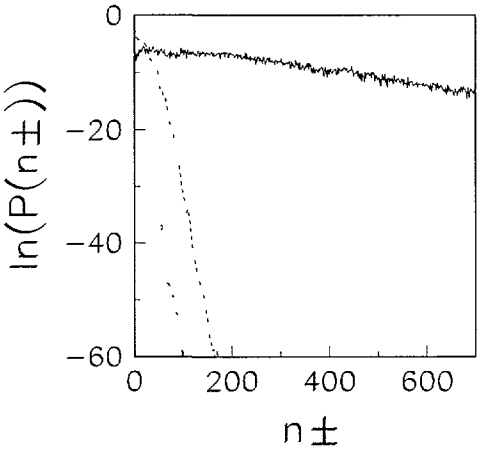

Fig. 11. — Probability distribution at $t = 1.6 \times 10^6$ as a function of $n_\pm = 2^{-1/2}(n \pm n')$ for the case of Figure 10: $P_+(n_+)$ (full line); $P_-(n_-)$ (dashed); dotted line is the distribution $P_+(n_+)$ for $U = 0$.

increase of κ the probability to have a distance between particles of the order of l_1 decreases as $1/\kappa(t) \sim 2/(l_1 \ln t)$ and therefore, we expect that the diffusion rate of the pair will decrease with time as $D_+ \sim D_{\rm ef}/\ln^\mu t$. Here $\mu = 1$ and $D_{\rm ef}$ is some effective "subdiffusion" rate. While the above probability argument gives $\mu = 1$ it is quite possible that sticking in the region with $\kappa \gg l_1$ will give a faster decrease of D_+ with a higher value of μ. As it was discussed in [6] the growth of pair size should also give logarithmic corrections to the coherent localization length in the quasi-1d case (1) ($l_c \sim {l_1}^2/\ln^\mu l_1$).

Another confirmation for the delocalization transition below one–particle threshold is given by the analysis of the number of effectively excited states. Indeed for $\nu = 3$ one has $\Delta N \approx$

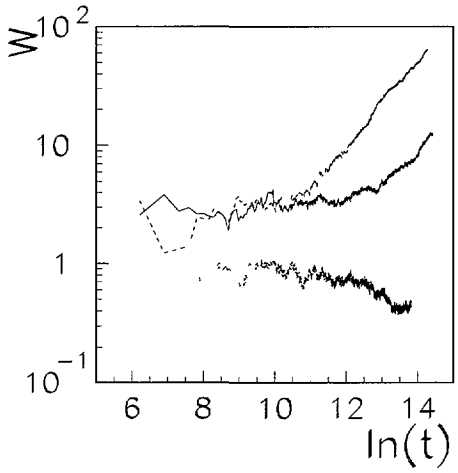

Fig. 12. — Dependence of $W = \Delta N/t = \sigma_+^2 \sigma_-^{1/2}/t$ on time for $\nu = 3$, $k = 0.7$, $\epsilon = 0.9$ and $U = 2$ (upper curve), $U = 1$ (middle curve), $U = 0.5$ (lower curve).

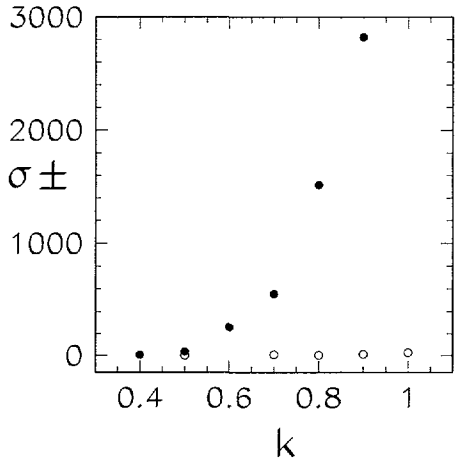

Fig. 13. — Dependence of σ_+ on k for $\nu = 3$ at $t = 2 \times 10^5$, $U = 2$ (full circles), $U = 0$ (open circles).

$\sigma_+^2 \sigma_-^{1/2}$ According to our data, for sufficiently strong interaction U the quantity $W = \Delta N/t$ grows approximately linearly with time (see Fig. 12) while for small U values W decreases with time. Contrary to the case $\nu = 2$ this indicates that TIP delocalization takes place for U values bigger than a critical $U_{cr} \approx 0.7$. Above this critical value the number of excited states at a given time grows when k approaches to one–particle delocalization border k_{cr} (Fig. 13).

4. The Effective Lloyd Model

We also studied the model (2) with the kick perturbation given by (5). In this case, the non interacting problem can be reduced to the Lloyd model with pseudo–random sites energies [12]. For $\nu = 2$ and $E = 0$ the one particle delocalization border is $k_{cr} \approx 0.46$ [12]. For TIP problem the behaviour of this model is similar to that of Section 2. The strong enhancement

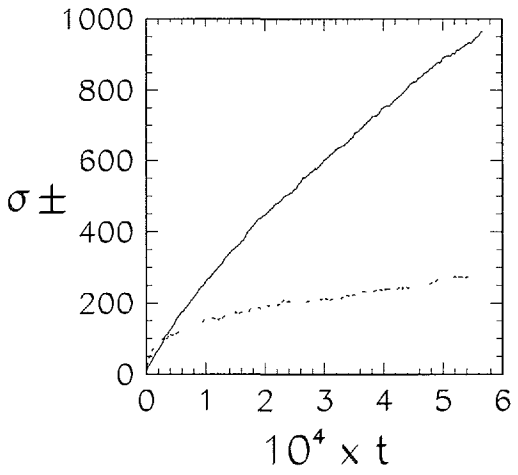

Fig. 14. — Dependence of second moments on time for the Lloyd model with $\nu = 2$, $k = 0.35$, $U = 2$; upper curve is σ_+, lower is σ_-. At $t = 0$ both particles are at $n = n' = 0$. Basis is $-256 < n, n' < 256$. For $U = 0$ one has $\sigma_+ \approx 16$.

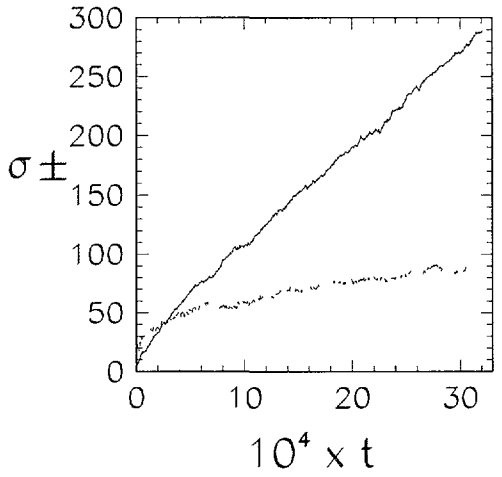

Fig. 15. — Dependence of second moments on time for the Lloyd model with $\nu = 3$, $k = 0.2$, $U = 2$; upper curve is σ_+, lower is σ_-. At $t = 0$ both particles are at $n = n' = 0$. Basis is $128 < n, n' < 128$. For $U = 0$ one has $\sigma_+ \approx 2$.

of propagation is demonstrated in Figure 14. Even if the investigation of this model is more difficult for the numerical simulations, our data indicate, as it was in Section 2, that for $\nu = 2$ the suppression of σ_+ is always present. In the same way we attribute this behaviour to the effective two dimensionality of the model $d_{\text{eff}} = 2$.

We also analyzed the tangent model (2), (5) for the case of three frequencies $\nu = 3$ ($V(\theta, \theta_1, \theta_2, \theta_3) = -2 \tan^- 1(2k(\cos\theta + \cos\theta_1 + \cos\theta_2 + \cos\theta_3))$). This case is similar to that discussed in Section 3. The behaviour of $\sigma_\pm$ is presented in Figure 15 and indicates the existence of delocalization transition for TIP below one particle delocalization border with $k_{\text{cr}} \approx 0.22$.

5. Conclusions and Discussions

Our numerical investigations definitely demonstrate the effect of enhancement of the localization length for TIP in a random potential. These results were obtained for kicked rotators models with frequency modulation. Such approach allows to model efficiently TIP problem in an effective dimension $d_{\rm eff} \geq 2$. Numerical data for these models confirm the theoretical expectations [4–6] that TIP delocalization in $d > 2$ is possible below one–particle delocalization border. In agreement with [6] we found TIP pair delocalization and, at the same time, a logarithmic growth of the pair size. We attribute this growth to the noise produced by the random potential. Indeed a pair propagating in a random potential sees different realizations of disorder which act like some effective noise. Such noise originates transitions which increase the distance between the two particles. Even if the amplitude of these transitions is exponentially decreasing with the two–particle distance, it gives rise to logarithmic growth of pair size with time. This in turn produces a subdiffusive pair propagation $(\Delta n_+)^2 \sim D_{\rm eff} t/\ln^\mu t$. We give arguments for $\mu = 1$, but it is possible that due to sticking in the region with large distance between particles one can have $\mu > 1$. Further investigations should be done in order to determine the exact value of μ. Another qualitative argument for the subdiffusive propagation is the tunneling between states in which the two particles are far from each other (at a distance $R \gg l_1$) and states in which particles stay within l_1 ($R \leq l_1$). These states are quasi-degenerate since the spectrum of delocalized states is embedded in the spectrum of localized ones. In this situation even an exponentially small overlapping between these two kinds of states becomes important and it can lead to a subdiffusive pair propagation. Further work should be done for a better understanding of the final spectrum structure and the eigenfunctions properties for TIP in 3d.

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