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Enhancement of localization length for two interacting kicked rotators

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Abstract. We study the effect of coherent propagation of two interacting particles in a disordered potential. The dependence of the enhancement factor for coherent localization length due to interaction is investigated numerically in the model of quantum chaos. The effect of interaction for two particles in many dimensions is also discussed.

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1. Introduction

The quantum localization of dynamical chaos has received a great deal of attention during the last few years [1, 2]. It has been understood that quantum interference effects lead to a suppression of diffusive spreading in the action space in spite of the chaotic dynamics of the corresponding classical model. An important consequence of this phenomenon is the exponential localization of quantum eigenfunctions over the unperturbed levels. A close correspondence has been established between this dynamical localization and the Anderson localization in a random potential for solid state systems [3]. One of the most studied models in this field is the kicked rotator model (KRM), which in the classical limit corresponds to the Chirikov standard map [4] (CSM), a common paradigm of classical chaos. Although the KRM seems to be at first glance a purely mathematical model, it has however found important applications for real physical systems, such as for example the process of microwave ionization of Rydberg atoms [5]. Another useful property of the KRM is that it can be studied very efficiently in numerical simulations allowing us to investigate its properties in great detail.

In any case, the KRM describes one-particle quantum dynamics and in many respects it is quite similar to one-particle localization in a quasi-one-dimensional random potential [6]. The latter problem has been intensively studied in solid state physics and it is well understood from the theoretical point of view. In contrast, the case of interacting particles is much more complicated and a clear theoretical picture is still lacking (see for instance the recent review [7]). Usually this problem is studied near the ground state and it is

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common lore that a repulsive interaction would result in even stronger localization [8, 9]. However, the recent investigation of two interacting particles in a quasi-one-dimensional random potential [10] has shown that, even in the case of repulsive particles, interaction leads to an enhancement of localization length and a coherent propagation of two particles on a distance l_c much larger than the one-particle localization length l_1. According to [10] the enhancement factor is given by :

$$\frac{l_c}{l_1} \sim l_1 M \frac{U^2}{32V^2} \tag{1}$$

where U is the strength of on-site interaction between two particles, V is the one-particle hopping element between nearest sites which determines the size of the one-particle energy band and M is the number of transverse channels. As is well known for $M > 1$ the one-particle localization length l_1 is proportional to M (for $M < l_1$) [11] so that according to (1) $l_c \propto M^3$. In (1) the intersite distance a is taken to be equal to one and energy is taken near the centre of the band so that $k_F \sim 1/a = 1$. The interaction U is considered to be less than or comparable with V. The equation (1) is valid in the regime when the enhancement factor is greater than one, $l_c/l_1 > 1$.

The physical reasons according to which two particles propagate together, forming an effective pair, can be understood in the following way [10]. Since the interaction couples only nearby sites (or on site) then the particles initially located at a distance $r \gg l_1$ have only exponentially small effective coupling with each other due to the exponential decay of localized one-particle eigenstates. For such types of states two interacting particles (TIP) remain localized in the radius l_1 near their initial positions. These states are localized and form the majority of all states. However, there are other states in which TIP are initially close to each other so that $r < l_1$. As follows from the previous case $r \gg l_1$, the initially close particles ($r < l_1$) cannot become separated at a distance much larger than l_1 (otherwise we would contradict the previous case $r \gg l_1$). Therefore, they always remain at a distance $r \sim l_1$. At this distance interaction between particles is important. Qualitatively, the motion of one particle with respect to the other produces some kind of noise on it. This noise causes destruction of interference effects which produce one-particle localization. The destruction of interference allows particles to propagate at a distance l_c which is much larger than the distance between particles l_1. This propagation can take place only for two particles since as soon as they become separated ($r \gg l_1$) they become localized. The above picture explains qualitatively why even repulsive particles can propagate together coherently at a large distance $l_c \gg l_1$. Of course, to be more precise one should use a more rigorous approach (see below) to take into account that the effective 'noise' can have discrete (1 or 2 dimensions) or continuous spectrum (3 dimensions) so that finally the pair motion is localized (1 or 2d) or delocalized (3d). In some sense the method of TIP propagation in a random potential is similar to the method used by Münchhausen to save himself from a swamp.

The functional dependence in (1) can be understood in the following way. The enhancement factor l_c/l_1 is proportional to the probability to mix one-particle states by interaction. In fact $l_c/l_1 \sim \Gamma\rho$ where $\Gamma \sim U_s{}^2\rho$ is the rate of transition and ρ is the density of coupled states [10]. The above relation between $l_\phi = l_c/l_1, \Gamma$ and ρ was derived for photonic localization in a molecular quasicontinuum [12] where l_ϕ measures the localization length in a number of one-photon transitions. The proportionality between l_ϕ and Γ is similar to the proportionality of l_ϕ to the diffusion rate D ($D \sim \Gamma$) which is well established for the KRM. In the TIP model the size of the transition is l_1, the coupling matrix element is $U_s \sim U/N^{3/2}$ [10] and $\Gamma \sim U^2/NV$, where $N = Ml_1$ is the number

of unperturbed components in a one-particle eigenstate. The density of coupled states is $\rho = 1/\Delta E \sim N^2/V$ so that the enhancement factor is $l_c/l_1 \sim (U_s/\Delta E)^2 \sim N(U/V)^2$ in agreement with (1). It is interesting to mention that such an estimate is quite similar to the derivation of statistical enhancement for weak interaction and parity violation in neutron–nucleus reactions discussed by Sushkov and Flambaum [13]. However in [13] the interaction, even enhanced, remains relatively weak giving only small corrections while in our case the enhanced interaction $U_{\text{eff}} \sim (Ml_1)^{1/2}U$ is not small and it leads to significant changes in the properties of the system. Another interesting method of the derivation of (1) was given recently by Imry [14] based on the Thouless block picture which allows one to analyse different dimensions. His approach also shows the enhancement of conductance on a scale smaller than l_c.

In [10] the result (1) was also obtained by the reduction of the TIP problem to one of superimposed band random matrices (SBRM). There, the interaction, even if repulsive, creates an effective thick wire along the diagonal $n_1 = n_2$ in the two-dimensional plane (n_1, n_2) of indices corresponding to two particles. The effective width of the wire (effective number of transverse channels) is determined by the number of levels Ml_1 coupled by interaction in the one-particle basis. Outside of this width interaction is exponentially small and can be neglected at least in the first approximation. The large number of effective transverse channels $M_{\text{eff}} = Ml_1$ leads to enhancement of localization length with the enhancement factor proportional to M_{eff}.

All the above approaches were based on the assumption of statistical independence of transition matrix elements and eigenenergies in the one-particle basis. This approximation seems to be reasonable due to randomness of the potential and finite radius of the interaction. However, it is very important to have a direct check and to verify the prediction (1). Some numerical checks were presented in [10]. Here we present the results of a more detailed numerical investigation which we carried out for the model of interacting kicked rotators which had been discussed in [10]. We also present numerical results for a model with finite radius of interparticle interaction.

The paper is constructed as follows. In section 2 we introduce the model and present the main results for on-site interacting kicked rotators. The case of finite radius of interaction is discussed in section 3. Conclusions and discussions of results are presented in section 4.

2. The 'on-site' interaction model

To investigate the effect of enhancement of localization length by interaction we used the model of two interacting kicked rotators introduced in [10]. The model represents two particles on a ring perturbed by kicks that are periodic in time. The evolution of the wavefunction ψ on one period of perturbation is described by the unitary operator (Floquet operator) :

$$\hat{S} = \mathrm{e}^{-\mathrm{i}T(\hat{n}_1^2+\hat{n}_2^2)/2+\mathrm{i}U\delta_{n_1,n_2}} \times \mathrm{e}^{-\mathrm{i}k(\cos\theta_1+\cos\theta_2)} \tag{2}$$

with $\hat{n}_{1,2} = -\mathrm{i}\partial/\partial\theta_{1,2}$. For $U = 0$ we have two noninteracting kicked rotators which had been intensively studied during the last few years [1, 2, 3]. The classical dynamics is chaotic and diffusive for the chaos parameter $K = kT > 1$ [4]. The diffusion rate is approximately $D = n^2/t = k^2/2$ for $K \gg 1$. Quantum interference effects lead to suppression of this diffusion for typical irrational values of $T/4\pi$ and to exponential localization of eigenstates so that the averaged probability distribution over unperturbed levels decays as $|\psi_n|^2 \approx \exp(-2\,|\,n - n_0\,|\,/l_1)/l_1$. The localization length in the region

of strong chaos is approximately given by $l_1 \approx D \approx k^2/2$. The quasiclassical regime corresponds to $k \gg 1, T \ll 1, kT =$ constant and $l_1 > 1$.

For $U \neq 0$ interaction between particles is switched on. Using the Bessel expansion, (2) can be written as :

$$\hat{S} = e^{-iT(\hat{n}_1^2+\hat{n}_2^2)/2+iU\delta n_1,n_2} \times \sum_{m_1,m_2} J_{n_1-m_1}(k) J_{n_2-m_2}(k)(-i)^{n_1+n_2-m_1-m_2} e^{im_1\theta_1+im_2\theta_2}. \quad (3)$$

Since $J_{n-m}(k)$ is exponentially small when $|n-m| > k$, at each iteration of $\hat{S}$ many states ($\sim 2k$) are coupled. Inter-particle interaction acts only when the two particles have the same momentum, namely when they occupy the same site on the momentum grid (on-site interaction, if we adopt the solid state terminology). Due to its presence in the exponent, the interaction U can only take values in the interval $(0, 2\pi)$. Due to interaction the two particles are able to propagate coherently at a distance l_c much larger than the original one-particle localization length l_1, as was anticipated in the introduction. Of course this can happen if they initially started within a distance $r < l_1$. Even if very close to the TIP problem in the 1d Anderson model [10], our model has three different features. Indeed no randomness is occurring here and the interaction is neither attractive nor repulsive. In addition the perturbation couples many levels at each iteration (kick).

The quantum dynamics was investigated in numerical simulations for symmetric configurations with an effective number of unperturbed levels from 1000 to 2000. Antisymmetric configurations of two particles do not feel the on-site interaction U and are not interesting. We iterated the quantum operator $\hat{S}$ starting from two particles initially at the same site for different parameter values. The spreading of the wavefunction in the 2d space (n_1, n_2) was studied through the second moments along the diagonal line $n_1 = n_2$:

$$\sigma_+(t) = \tfrac{1}{4}\langle(|n_1| + |n_2|)^2\rangle_t$$

and across it

$$\sigma_-(t) = \langle(|n_1| - |n_2|)^2\rangle_t$$

as a function of the iteration time t. In any investigated case σ_+ was observed to saturate at a higher value than in the absence of interaction, see figure 1. On the other side σ_- keeps the same order of magnitude as l_1^2 (as it should in the absence of interaction). This means that the localization length is strongly enhanced along the diagonal $n_1 = n_2$ while it remains localized, with roughly the same localization length, across the diagonal. This is even more evident if one looks at the probability distribution $P(n_1, n_2) = |\psi(n_1, n_2)|^2$ at a fixed time $t \gg t^*$, where $t^* \approx l_1$ is the localization time, see for instance figure 2. In this diagram a local averaged distribution function is represented in the quarter of space $n_1, n_2 > 0$, in a semilog plot. The channel of propagation along the diagonal $n_1 = n_2$ is manifested in the contour lines drawn at the surface basis.

From the distribution function important information can be extracted by computing the following distributions :

$$P_\pm(n_\pm) = \sum_{|n_1 \pm n_2| = n} |\psi(n_1, n_2)|^2$$

represented in figure 3 as a function of $n_\pm = |n_1 \pm n_2|/\sqrt{2} = n/\sqrt{2}$. These distributions give a measure of the 'perturbed' localization lengths along (+) and across (−) the principal diagonal. It is relatively easy to derive from them the respective localization lengths $l^\pm$ by the usual best fitting procedure. Indeed the distributions are quite close to exponential curves $P_\pm \sim 2\exp(-2n_\pm/l^\pm)/l^\pm$ as can be inferred from figure 3. In the same diagram we show the probability distribution in the absence of interaction $P_+^0 = 8n_+ \exp(-2^{3/2}n_+/l_1)/{l_1}^2$

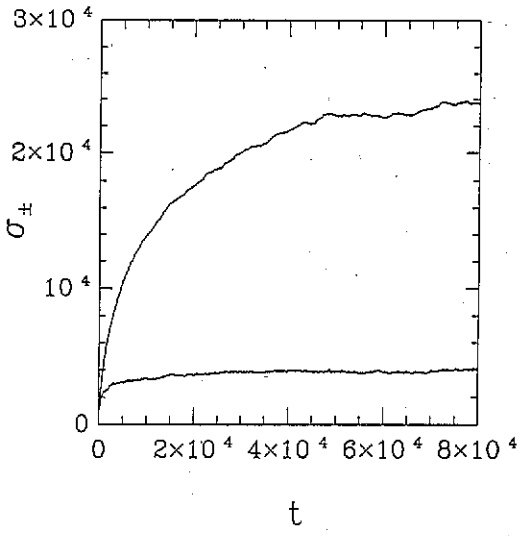

Figure 1. Dependence of second moments on time in model (1.1) with $k = 7$, $K = kT = 5$, $U = 2$; upper curve is σ_+, lower is σ_-. At $t = 0$ both particles are at $n_1 = n_2 = 0$, basis is $-800 \leqslant n \leqslant 800$. For $U = 0$ $\sigma_+(t) \approx 600$ for large t.

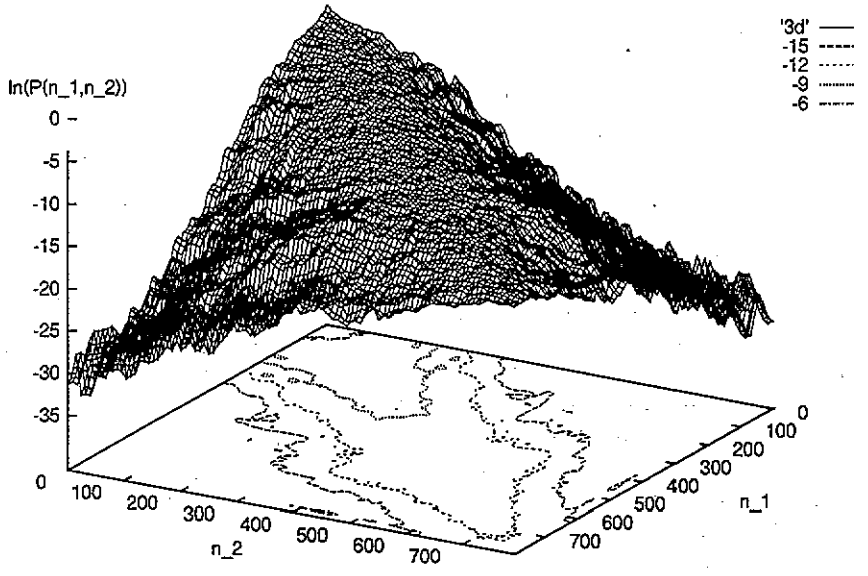

Figure 2. Probability distribution for two particles in the case of figure 1 at $t = 8 \times 10^4$. Different contours show different probability levels.

with $l_1 = k^2/2$. This noninteracting distribution is quite similar to P_-. The localization lengths $l^{\pm}$ are then plotted versus $l_1 = k^2/2$ in order to check the validity of (1). For sake of comparison the lines with power 1 and 2 are drawn. The dependence of coherent localization length $l_c = l^+$ on the one-particle length l_1 can be satisfactory described by $l^+ \approx 0.5{l_1}^2$ at $U = 2$ while $l^- \approx 1.5l_1$. However, the least-squares fit for the data of figure 4 gives $l^{\pm} \sim {l_1}^{\alpha_{\pm}}$ with $\alpha_+ = 1.44 \pm 0.29$ and $\alpha_- = 1.14 \pm 0.07$. We attribute the difference from the theoretical values 2 and 1 to the insufficiently large interval of variation of l_1 (only

4 times). Furthermore, detailed numerical investigations should be done to extract more accurate values of $\alpha_\pm$. Another interesting point following from figures 1–4 is that for the same length l_1 the coherent length l_c is significantly larger than in TIP in the 1d Anderson model considered in [10]. This can be seen by direct comparison of σ_+ values. One of the reasons for this difference could be the different type of hopping in the KRM where one kick couples many levels.

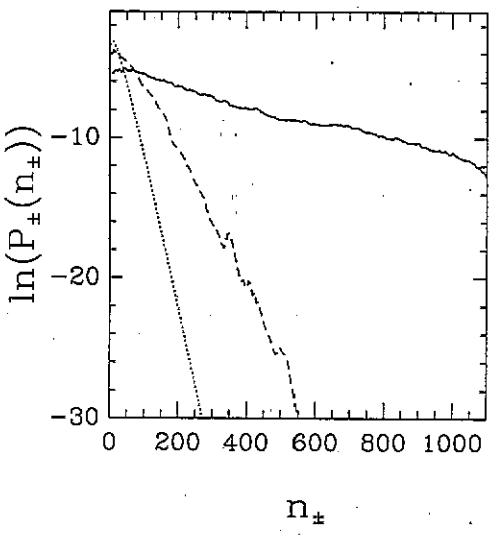

Figure 3. Probability distribution as a function of $n_\pm = 2^{-1/2}(n_1 \pm n_2)$ for the case of figure 2: $P_+(n_+)$ (full line); $P_-(n_-)$ (dashed); the dotted line is the theoretical distribution $P_+^0(n_+)$ for $U = 0$.

To determine the numerical factor in the dependence of l_c on both U and l_1 one should also study the problem at small values of $U \ll 2$. However, here for observation of the enhancement l_c/l_1 one should work at much larger values of l_1 than we used in figures 1–4. This requires a sharp increase of the basis and makes the numerical calculations too difficult. The dependence on U has been investigated in the following way. According to (1) we expect that there should exist a critical U_{cr} given by $U_{cr}\sqrt{l_1} > C$ with $C \sim 1$. To check this we consider the same model but with random rotating phases, which means that $T(n_1^2 + n_2^2)/2$ in the first exponent is replaced by $f(n_1) + f(n_2)$, with $f(n) = f(-n)$ a random function in the interval $(0, 2\pi)$.

In this way we can change configuration by varying the random realization and obtaining results for the average behaviour. The results averaged over 10 realizations of disorder are presented in figure 5. The asymptotic value reached by the second moments $\sigma_\pm^\infty = \lim_{t\to\infty} \sigma_\pm(t)$ are plotted in units of the same value in the absence of interaction ($U = 0$). Error bars are due to fluctuations in varying the random configuration. For small U, σ_+^∞ and σ_-^∞ are both increasing up to double their value without interaction. When $U > U_{cr}$ full (σ_+) and open (σ_-) circles start to deviate from each other thus indicating the presence of a sharp transition. In our case the transition starts at one particle localization length $l_1 = 8$ which approximately agrees with the observed critical value $U_{cr} \approx 0.3$ and $C \approx 1$. We were not able to extract more precise information on the dependence of l_c on U due to the heaviness of numerical simulations.

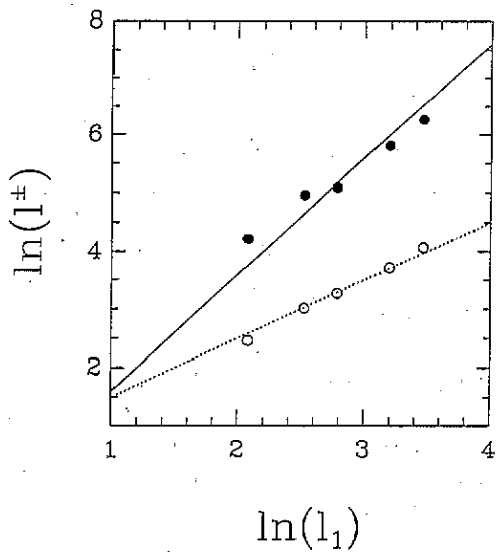

Figure 4. Dependence of localization length l^+ (full circles) and l^- (open circles) on one-particle localization length $l_1 = k^2/2$ for $K = 5$, $U = 2$ and $4 \leqslant k \leqslant 8$. Full line shows dependence $l_c = l^+ \propto l_1^2$, dashed line marks $l^- \propto l_1$.

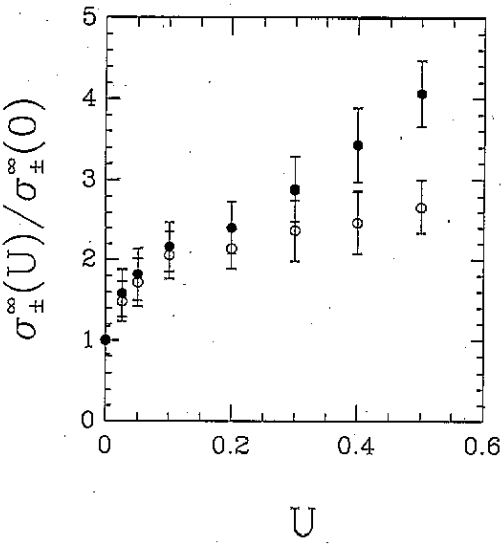

Figure 5. Dependence of enhancement for $\sigma_\pm^\infty$ on U for the model (2.1) with random rotation phases (see section 2), $k = 4$. Error bars are obtained from $\sigma_\pm^\infty$ for 10 different realizations of disorder.

3. The model with finite radius of interaction

In this section we analysed the effect of a finite range interaction on the dynamics. To be more precise we chose in (2), instead of the former on-site interaction $U\delta_{n_1,n_2}$ a more general, finite radius interaction :

$$U\eta(n_1, n_2)\theta(b - |n_1 - n_2|)$$

where $\theta(x)$ is the usual step function which is zero for $x < 0$ and one for $x \geqslant 0$. The phase η is a random number in the interval $(-1, 1)$ which depends only on n_1 if $n_1 < n_2$ and only on n_2 if $n_2 < n_1$. It is quite clear that for $b = 0$ it becomes the previous one with diagonal disorder. The diagonal disorder creates some difference from the model of section 2 since now the interaction depends not only on the difference $n_1 - n_2$. However, physically it is clear that diagonal disorder in the interaction will not greatly change the results. Indeed, the main point is to have *some* coupling between two particles and the sign of interaction is not very important for the destruction of interference, since the one-particle random potential is already acting. Our numerical results confirm that disordered one-site interaction gives qualitatively the same effects as for on-site interaction $U\delta_{n_1,n_2}$. We usually investigated the cases with different interaction radius $R = 2b + 1$ and $U = \pi$.

Our main interest is to investigate the effect of interaction with finite radius R. From the theoretical point of view we can expect that for interaction radius $R < l_1$ equation (1) is still valid since the particles are effectively coupled at a distance l_1. However, for $R > l_1$ the size of the effective thick wire on the lattice n_1, n_2 is defined by R so we can expect that the enhancement factor will become larger $l_c/l_1 \sim (R + l_1)$. This expression should remains valid up to values of $R \ll l_{12}$ where l_{12} is the 2d localization length for infinite radius R: $\ln l_{12} \sim l_1$. Indeed, for $R \gg l_{12}$ with the chosen type of interaction one should have the same localization length as in 2d. To avoid misunderstanding, let us mention that in our model the interaction enters the phase of the evolution operator and a large interaction radius does not lead to divergence of physical characteristics.

The results of our numerical simulations for finite interaction radius R are presented in figures 6–9. In figure 6 and figure 7 we show σ_+ and σ_- for three different b values ($b = 0, 4, 16$) which roughly agree with the above estimates. In agreement with the above picture the enhancement factor remains practically unchanged for $R < l_1$. Only the case $b = 16$ has $R > l_1$ and this produces a significant growth in σ_+ (and even in σ_-). The distributions $P_\pm(n)$, as defined in the previous section, are shown in figure 8 and demonstrate a sharp increase of l^+ compared to $U = 0$. In the same way we took the asymptotic values $\sigma_\pm^\infty$ reached by $\sigma_\pm(t)$ at large time t and we plot in figure 9 their square root as a function of the radius of interaction R (the values of $l^\pm$ have a similar behaviour). This figure confirms the above arguments that the enhancement starts to grow only for $R > l_1$. However, it should be mentioned that the increase of R leads to a growth not only of σ_+ but also of σ_-. Indeed, σ_+ and σ_- are growing in the same way: from the same figure 9 one can see that the ratio $\sigma_+^\infty/\sigma_-^\infty$ is approximately constant as a function of the interaction radius R. The physical explanation of this similar growth is quite simple: the increase of R leads not only to the increase of the coherent propagation length but also to an increase of the effective size of the pair which becomes of the order of $R \gg l_1$. Unfortunately, we were not able to study numerically the regime $R \gg l_1$ (in our case the maximal ratio $R/l_1 \approx 2$) and it was not possible to check the dependence $l_c \sim Rl_1$.

4. Conclusions and discussions

Above we presented the results of our numerical investigation about two interacting kicked rotators in the domain of quantum chaos. They clearly demonstrate that on-site interaction between two rotators in momentum space leads to a large enhancement of localization length if compared with the noninteracting case (figures 1–3). The localization length for coherent propagation of two particles $l_c = l^+$ is significantly larger than the distance between them $l^- \approx l_1$. The maximal ratio l^+/l^- in our numerical simulations was close to 10 (figure 4) which justifies the fact of effective enhancement of localization length for

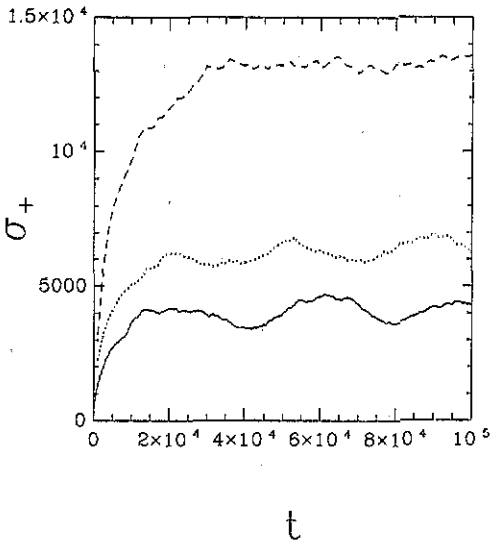

Figure 6. Dependence of σ_+ on time for the model with finite interaction radius; $k = 5.7$, $K = 5$, $U = \pi$; $R = 1$ (full curve), 9 (dotted), 33 (dashed). Initial conditions are as in figure 1, basis is $-500 \leqslant n \leqslant 500$. The noninteracting case $U = 0$ has $\sigma_+ \approx 250 \approx l_1^2$ (see figure 3 in [10]).

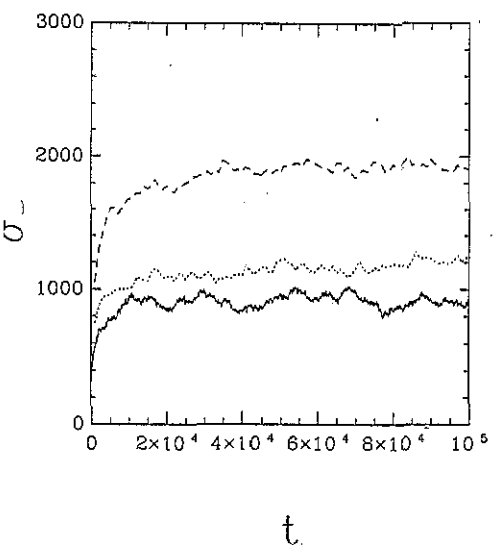

Figure 7. Same as in figure 6 but for σ_-.

coherent propagation of two particles. The direct check of the relation (1) shows that the coherent localization length $l_c = l^+$ grows approximately as $l^+ \sim l_1^2$ but more detailed numerical calculations are necessary to have a more accurate check of the power (see also the discussion below).

Another part of our investigations was devoted to the effects of a finite radius of interaction R between particles. They definitely show that for $R < l_1$ the enhancement is not sensitive to the value of R (figure 9). The physical reason is quite clear. Indeed, on-site interaction couples one-particle states in a radius of l_1 and therefore interaction with

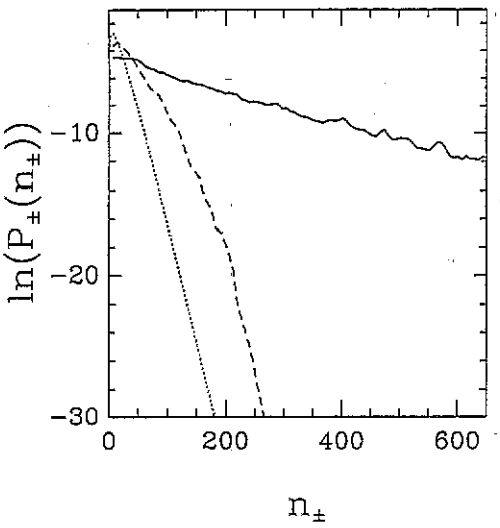

Figure 8. Probability distribution as a function of $n_\pm = 2^{-1/2}(n_1 \pm n_2)$ for the case of figure 6 and $R = 9$: $P_+(n_+)$ (full line), $P_-(n_-)$ (dashed line). $P_+^{(0)}(n_+)$ (dotted line) is the theoretical distribution for $U = 0$.

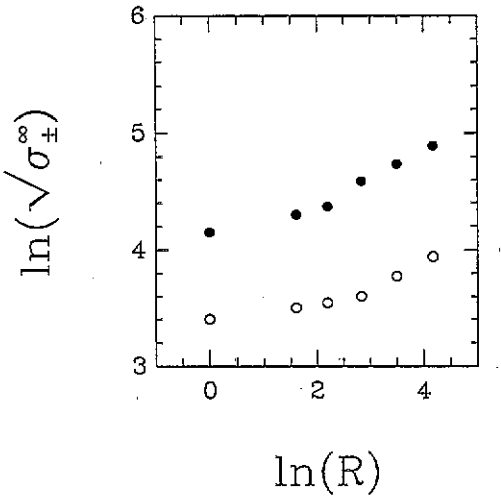

Figure 9. Dependence of σ_+^∞ (full circles) and σ_-^∞ (open circles) on interaction radius R; $k = 5.7$, $K = 5$, $U = \pi$.

$R < l_1$ does not give significant changes. For $R \gg l_1$ the enhancement factor starts to grow with R. One can expect that in the regime $R \gg l_1$ the radius R will play the role of number of coupled states $Ml_1 = R$ in an effective thick wire so that $l_c/l_1 \sim R$. Of course, this growth can continue only up to $R < l_{12}$ where l_{12} is the one-particle localization length in two dimensions and $\ln l_{12} \gg 1$. While our results definitely show an increase of enhancement with R the power of growth is around 0.25 and is significantly less than 1. We attribute this difference to the fact that the ratio R/l_1 was not big enough (in figure 9 $R/l_1 < 2.1$) and the asymptotic regime had not yet been reached. Further increase of R is

quite difficult since l_c becomes comparable with the size of the basis.

In general our results confirm the relation (1) but a more detailed verification of this equation is still desirable.

Let us now discuss in more detail the different consequences of the result (1). First we start from different dimensions d. For $d = 2$ the length l_1 in (1) should be understood as the one-particle localization length in 2 dimensions. The number of transverse channels M is approximately equal to l_1 so that finally $l_c \sim {l_1}^3$. For dimension $d = 3$ an interesting situation appears below the Anderson transition for one particle [15]. Indeed, it is possible to realize a random potential in which *all* one-particle eigenstates are localized for the hopping strength $V < V_c$ (a shift of mobility edge by interaction is not a very interesting case). As a typical example let us consider the Lloyd model with diagonal disorder $E_{n_1,n_2,n_3} = \tan \phi_{n_1,n_2,n_3}$ and hopping V on a cubic lattice, where ϕ_{n_1,n_2,n_3} are random phases homogeneously distributed in the interval $[0, \pi]$. In this case $V_c \approx 0.2$ and below this value *all* states are localized. For two interacting particles in such random potential the effective strength of hopping for a pair will be strongly enhanced $V_{\text{eff}} \sim \sqrt{N}U$. Here U is an on-site (or nearby site) interaction and $N \sim {l_1}^3$ is the effective number of states coupled by interaction. Since l_1 can be quite large near to (but below) the one-particle transition point V_c, then two particles, even if characterized by repulsive interaction, can be delocalized when *all* one-particle eigenstates are exponentially localized [15]. Another way to see this effect is to say that the pair feels the disorder averaged over the size of the pair l_1 which gives a strong effective decrease of disorder. Since in 3d delocalization takes place for $V_{\text{eff}} > V_c$ generally there is no requirement to have $l_1 \gg 1$ and it is not necessary to take V very close to V_c. The condition $V_{\text{eff}} > V_c$ gives the boundary of pair delocalization $U{l_1}^{3/2}/V > 1$.

The appearance of delocalization for a pair in 3d leads to quite interesting properties of the energy spectrum. Indeed, for particles located at a distance $r \gg l_1$ from each other the effective interaction is exponentially small ($\sim \exp(-2r/l_1)$) due to the small overlapping of one-particle eigenstates. Therefore, such states remain localized while the delocalization will take place only for the states with interparticle distance $r < l_1$. Since the localized states with $r \gg l_1$ form an everywhere dense spectrum this would mean that the continuous spectrum, corresponding to a delocalized pair, is *embedded* into the pure point spectrum of almost noninteracting one-particle states.

The above construction of the spectrum cannot be considered as the final one. In fact, it is only the first approximation since, generally speaking, the mixed spectrum is unstable with respect to small coupling between quasi-degenerate levels. In the present case the coupling is exponentially small but nevertheless it can change in principle the structure of the spectrum. The physical reason for such a possible change can be understood in the following way. The delocalized pair propagates in a random potential which acts as some effective noise. This can increase the size of the pair even if the matrix elements for transitions with $r = n_- \gg l_1$ are exponentially small. Due to this noise the size of the pair will grow in time. The rate of growth can be estimated as $D_- = {n_-}^2/t \sim {l_1}^2 \Gamma \exp(-2n_-/l_1)$ with $\Gamma \sim U^2/{l_1}^3 V$. This gives a logarithmically slow growth of the pair size $n_- \sim (l_1/2) \ln t$. At the moment it is not quite clear what will be the effect of the pair size growth on pair propagation in n_+. At minimum, the displacement of the pair should become slower than diffusive ${n_+}^2 \approx (n_1 - n_2)^2 \sim t/\ln t$. However, it is quite possible that sticking at $n_- \gg l_1$ will produce a more significant effect on the growth of n_+ since in the region $n_- \gg l_1$ the matrix elements for transitions in n_+ are also exponentially small. It is interesting to note that even in the case of strong attraction between particles the coupled state should be destroyed during the propagation in a random potential. Indeed, during the displacement of

the pair, disorder leads to transitions from the coupled state to the continuum, leading to the destruction of the pair. Usually, the destruction rate is proportional to the squared amplitude of the disorder and this can make the lifetime of coupled state relatively short. In contrast to this case, the effective life time of a pair of repulsive particles discussed above can be much larger since n_- grows only logarithmically with time. In some sense the interference creates exponentially high barriers which effectively push particles to stay together. In the quasi-one-dimensional case with $l_1 \gg 1$ the effects of slow pair size growth can also lead to the appearance of logarithmic corrections in the expression for the enhancement factor in (1). For example, we expect $l_c/l_1 \sim l_1/(\ln l_1)^\nu$ with $\nu \sim 1$ for $M \sim 1$. The effects of TIP in 3d systems below the Anderson transition when all one-particle eigenstates are localized are quite interesting and, at present, we try to study them in numerical simulations with effective 3d models [16]. The conclusion about TIP delocalization below the one-particle Anderson transition was also made in [14].

Up to now we have discussed the effects of interaction for only two particles. However, for solid state systems the natural question is: what will happen for a finite particle density ρ_e? As was discussed in [10], the above picture of TIP can be quite useful in the regime of small density $l_1 \ll 1/\rho_e \ll l_c$. In this case the interaction is mainly reduced to interaction between two isolated particles. If all the particles are separated from each other by a distance $L \sim 1/\rho_e \gg l_1$ then the interaction is exponentially small, all particles are localized and the current through such a sample is exponentially small. However, it is possible to have another type of configuration when the particles are distributed by pairs of size l_1. In this case pairs can easily propagate at a distance $l_c \gg L \sim 1/\rho_e \gg l_1$. Collisions of pairs will go in a random way and will destroy interference effects for a pair. These collisions will lead to delocalization and appearance of finite conductivity in an infinite system. It is interesting to note that it is enough to have only one pair when all other particles are well separated by the distance $L \gg l_1$. Then the collisions will allow transfer of the charge through the whole sample. However, the above consideration, based on (1) and being correct for particle energy at the centre of the band ($E \sim V$), should be applied more accurately for low energies near the ground state. Indeed, as was discussed in [10], at low energies one should consider a transition from a lattice to a continuous system in which the enhancement factor should be proportional to $l_c/l_1 \sim (k_F l_1)M$ since $k_F l_1$ determines the number of independent components in a localized state (for $M \gg 1$ the factor M should be replaced by $k_F a_t$ where a_t is the transverse width of the sample). Near the ground state $\rho_e \sim k_F$ and it seems that the condition $l_1 \ll 1/\rho_e \sim 1/k_F$ implies that the enhancement does not work at low energies. Due to this, at small densities there is no formal contradiction with the results of [8], according to which repulsive interaction reduces the localization length near the ground state. To gain a better understanding of the situation at small ρ_e a more exact analysis should be carried out to obtain a more precise expression for l_c in the continuous limit. In principle, the average difference of energies for two repelling particles ($k_F \sim 1/a = 1$) at a distance $r \gg l_1$ (E_∞) and $r < l_1$ (E_{l_1}) is of the order of $|E_\infty - E_{l_1}| \sim U/l_1$ and is not very large for large l_1. In fact this difference is even less than the amplitude of disorder W (we take the case of the 1d Anderson model discussed in [10] with diagonal disorder in the interval $\pm W$ where near the centre of the band $l_1 \approx 25(V/W)^2$). Finally, even for energies near the ground state two excited particles with $E \sim V$ should propagate at a distance much larger than l_1 and this effect should be also seen by the renormalization methods used in [8, 9]. It is possible that for investigation of the continuous limit $k_F a \ll 1$ at low energy the approach used in [17] for two particles with strong attraction can be useful after some extension.

The most interesting case with density $\rho_e \sim 1$ formally cannot be analysed on the basis

of the result (1) for TIP. In this case as a first approximation one can consider interaction of two excited quasi-particles when all others simply form the Fermi sea. Then the interaction between these two quasi-particles can be studied in the same way as for TIP. Since the density ρ_e is large the quasi-particles have the wavevector $k_q \sim k_F \sim 1/a = 1$ and therefore the enhancement factor is large. As a consequence, at small density of quasi-particles ρ_q ($l_1 \ll 1/\rho_q \ll l_c \sim l_1^2$) the quasi-particle pairs can propagate on a large distance and we can expect that the conductivity will not be exponentially small. In 3d for a 'gas' of quasi-particles the possible slow growth of the pair size should be less important since collisions between pairs give rise to destruction of interference and finite conductivity in the regime where all quasi-particles are localized. Due to the existence of an exact connection between localization in 1d and 1d disordered spin systems [9] it would be interesting to understand possible manifestations of the analogue of two-particle interaction for spin systems.

Finally, let us briefly discuss the possibilities of application of the observed enhancement for explanation of large persistent currents observed in experiments with small metallic rings [18]. Formally the coherent localization length (1) is strongly enhanced in the presence of interaction. Nevertheless, the direct estimates for the model of interacting kicked rotators and numerical results (see figure 1 and [10]) clearly show that the diffusion rate on the time scale $l_1 \ll t \ll l_c$ is not larger than the classical rate at $t \ll l_1$. In the optimal case $U \sim 1$ we can have the diffusion rate for a TIP pair $D_2 \sim D_1 \approx k^2/2$. It follows that the time to cross a sample will not be decreased by interaction. However, the magnitude of persistent current depends not only on the diffusion rate but also on the density of levels which in principle can become very large for multi-particle systems. As was remarked in [10], one can expect that the enhancement of conductance in interacting systems should be proportional to the enhancement factor l_c/l_1. The approach developed by Imry [14] directly shows that the high density spectrum of TIP states leads to such enhancement of conductance on scales smaller than l_c. Therefore, the possibility of enhancement of persistent current due to interaction is still open and should be studied in more detail.

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