GEOMETRIC STRUCTURE AND CONVEX DECOMPOSITION OF U-POLYGONS

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In a given space Y, the uniqueness problem of Geometric Tomography asks for the minimum number of radiographies needed for the unique determination of a geometric object in Y. Usually the problem is addressed to the class $\mathcal{C}_Y$ of all convex bodies of Y. A crucial step is to highlight geometric structures linked to possible ambiguities in the reconstruction process.

If radiographies are performed through parallel X-rays in a set U of directions, the so called *U-polygons* come out, which are the convex counterpart of more general structures called *switching components*. Understanding their geometric structure often allow uniqueness results to be achieved. There are some geometric parameters which can help in such an investigation. For instance, in the Euclidean plane $\mathbb{R}^2$, no U-polygons exist if U consists of 4 directions with transcendental cross-ratio ρ. This means that such a set of four directions uniquely determines a convex body in $\mathcal{C}_{\mathbb{R}^2}$ ([4]). The same is true for the class $\mathcal{C}_{\mathbb{Z}^2}$ if $|U| = 7$, or $|U| = 4$ and $\rho \notin \{2, 3, 4\}$, up to a reordering of the directions (see [3], and also [2] where a further geometric parameter is considered, namely the *class* of a U-polygon). Also, the presence of symmetry seems to be deeply related to the spread of ambiguities in the reconstruction processes. In this direction an important result has been achieved in [5], where switching components are characterized as the linear combination of switching element. When U-polygons are considered the role of convexity does not appear immediately. This is obtained in [1], where an alternative geometric approach leads to a kind of convex counterpart of the above characterization

References

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