Combinatorial Geometry Summer School "G. Tallini"

Brescia, Italy, 8-13 July 2002

Organized by:

MIUR Cofin 2001 national research group "Strutture Geometriche, Combinatoria e loro Applicazioni"

Seminario Matematico di Brescia

Università Cattolica, sede di Brescia, Dipartimento di Matematica

Politecnico di Milano, Dipartimento di Matematica

Book of Abstracts

July 2, 2002

`http://www.dmf.unicatt.it/summerschool/`

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Contents

Schedule of courses

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
9.15-10.00	De Clerck	De Clerck	Ceccherini	Zagaglia	De Clerck	Ceccherini
10.00-10.45	De Clerck	De Clerck	Ceccherini	Zagaglia	De Clerck	Ceccherini
	Coffee break					
11.15-12.00	Zagaglia	Maffioli	Bonisoli	De Clerck	Bonisoli	Bonisoli
12.00-12.45	Zagaglia	Maffioli	Bonisoli	De Clerck	Bonisoli	Bonisoli
	Lunch					
15.00-15.45	Bonisoli	Maffioli	Lucchetti	Meloni	Meloni	
15.45-16.30	Bonisoli	Maffioli	Lucchetti	Meloni	Meloni	

Permutations, Planes, Perspectivities

Arrigo Bonisoli
Univ. Modena e Reggio Emilia, Italy

1. Are finite projective planes geometric or combinatorial objects?
2. A review of permutation groups and sets.
3. Collineation groups: some classics and the role of perspectivities.
4. An exercise: a non–classical representation of a classical group.
5. Some recent results on collineation groups fixing an oval.
6. Benz-geometries with many symmetries.
7. Computer algebra packages may be of help.
8. Sets of involutory permutations, Buekenhout ovals and graphs.

This course has the scope of emphasizing some meaningful links between classical results in the theory of finite projective planes and current research in various related areas. Although some of the famous problems remain essentially open and probably as hard as ever, reasonable progress is recorded in areas where the interplay between geometry and combinatorics had not been fully exploited before. The focus will be on showing the role of different contributions, rather than giving proofs in full details. A basic knowledge of finite projective planes and permutation groups will be assumed: if one has time to read or browse something before the School starts, I suggest the classical textbook [1].

References

[1] D.R. Hughes and F. Piper, *Projective planes*, Springer, 1970.

Game Theory and Incidence Structure

Pier Vittorio Ceccherini
Univ. di Roma "La Sapienza", Italy

1. Simple games: basic terminology and examples.
2. The dual game. The algebra of simple games.
3. Pregames and weigthed graphs
4. Vector-weighted simple games and dimension theory. The game behind a simple game.
5. General and pairwise trading: weighted games and linear games
6. Incidence structures, geometry and simple games.

The course is an introduction to the theory of simple games, as developed in [1]. Some connections with graphs and projective spaces will also be considered.

References

[1] A.D. Taylor and W.S. Zwicker, *Simple games*. Princeton University Press, 1999.

Games in Extensive Form

Roberto Lucchetti
Politecnico di Milano, Italy

1. Definitions and examples.
2. The theorem of Zermelo.

(α, β)-Geometries from Polar Spaces

Frank De Clerck
Univ. Ghent, Belgio

Partial geometries and semipartial geometries are special classes of (α, β)-geometries. In the summer school on Finite Geometries, (Potenza 1-12 September 1997) I gave an outline of constructions of partial and semipartial geometries.
(see `http://cage.rug.ac.be/~fdc/fdceng.html` for the lecture notes)

In the meantime a lot of new results are known and some of the theory has been extended for general (α, β)-geometries. In the lectures we will especially focus on those (α, β)-geometries that are related to polar spaces and we will treat the following topics.

1. What is a polar space? What is an (α, β)-geometry? What is the relation?
2. Examples and characterizations of strongly regular (α, β)-geometries.
3. Construction of (α, β)-geometries from other ones.
4. Embedding of (α, β)-geometries in projective and affine spaces.

Each topic will be covered in more or less two lectures.

References

[1] F. De Clerck and H. Van Maldeghem, *Some classes of rank 2 geometries*, chapter 10 of Handbook of Incidence Geometries (ed. F. Buekenhout), Elsevier 1995.

[2] F. De Clerck, *Partial and semipartial geometries: an update* (see `http://cage.rug.ac.be/~fdc/fdceng.html` for a preprint)

[3] S. E. Payne and J. A. Thas, *Finite Generalized Quadrangles*, volume 110 of Research Notes in Mathematics, Pitman, Boston, 1984.

Combinatorial Optimization and NP-Completeness: an Introduction

Francesco Maffioli
Politecnico di Milano, Italy

The theory of NP-completeness has been developing since 1971 and has become a key ingredient of all approaches to solve difficult Combinatorial Optimisation Problems (COP). This talk aims at introducing both the fundamentals as well as some of the recent advances in this field.

1. What's a Combinatorial Optimisation Problems (COP) ?
2. Recognition and optimisation problems.
3. Tractability and polynomiality.
4. The classes P, NP, co-NP and the P$\subset$NP conjecture.
5. Polynomial reductions: NP-complete and NP-hard problems.
6. The polynomial hierarchy.
7. How to cope with NP-hard COP's.
8. Approximability classes: PAS, FPAS, APX.
9. The PCP conjecture and its role in proving non-approximability results.
10. Basic notions of Parametrised Complexity.

Species and Types in the sense of Andrè Joyal

Giancarlo Meloni
Univ. degli Studi, Milano, Italy

Combinatorial species. Structures and types.
Fundamental operations.
Generating series for cardinality, for types, and for cycles.
Reduction of types to species. Permutational species and plethism.
Weighed species. Multisorted species. Virtual species. Linear species.
Relations between Redfield-Pólya (1927-1937) and André Joyal (1981) theories, for counting types.
Examples.

References

[1] A. Joyal, *Une thorie combinatoire des sries formelles*, Advances in Mathematics,42, 1981, 1-82.

[2] F. Bergeron - G. Labelle - P. Leroux, *Combinatorial Species and Tree-like Structures*, Cambridge University Press, 1998.

Some Topics in Matroid Theory

Norma Zagaglia Salvi
Politecnico di Milano, Italy

1. Matroids arising from graphs, matrices, vector spaces, transversals.
2. Examples: uniform, graphic, cographic, representable, algebraic, transversal matroids. The Fano matroid.
3. Independent sets and bases. Rank function. Closure operation. Circuits. Axioms systems for a matroid in terms of these concepts.
4. Matroids and lattices.
5. Duality in graph theory and in matroids.
6. Operations to a matroid: deletion and contraction. Minors.
7. Connectivity.

References

[1] J. G. Oxley, *Matroid Theory*, Oxford University Press, 2001.

[2] J. G. Oxley, *On the interplay between graphs and matroids*, Survey in Combinatorics, 2001, ed. J. W. P. Hirschfeld, Cambridge University Press, Cambridge, (2001), 199 - 239.

[3] D. J. A. Welsh, *Matroid Theory*, Academic Press, 1976.

[4] R. J. Wilson, *An introduction to matroid theory*, Amer. Math. Monthly 80 (1973), 500 - 525.