Summer School in Brescia, Italy 30. June 2004
Monday 5 - Saturday 10 July 2004
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New developments in the theory of Linear Integer Programming

Abstract

In the area of *Mathematical Programming* (i.e. *Optimization* and *Operations Research*) there are developped methods for solving all kind of optimization problems occuring in industry or enonometry. These problems are often of the form: *Maximize* or *Minimize* a (real valued) function $f(x)$ under a collection of constraints which have to be satisfied by a vector x. Depending on the structure of f and the structure of the restrictions, one distinguishes in *linear*, *nonlinear*, *integer* or *combinatorial* optimization.

The aim of my lectures is to introduce some new developments in the theory of *Linear Integer Programming* which are mainly based on work of Conti and Traverso (1991), Sturmfels (1995), Thomas (1995), and some others (see the literature). The crucial point is that methods from Computer Algebra can be applied to solve linear integer programs; the fundamental link being *Gröbner bases of toric ideals* and *test sets for (families of) integer programs*.

List of Topics

1. *From Linear Algebra to Integer Programming*
   Linear restrictions, standard- and canonical form, feasibility, structure of the set of feasible points, Hilbert bases, Hermitean normal form, complexity status of the underlying desicion problems

2. *Ideals in Polynomial Algebras*
   Hilbert's Basissatz, Dickson's lemma, monomial ideals, binomial ideals, toric ideals

3. *Test Sets and Orderings*
   A-orders and term orders, integer programs with variable right hand side, generic optimization with test sets, existence, termination, compatible term orders, theorem of Conti and Traverso

4. *Test Sets and Gröbner Bases*
   Division algorithm, reduced Gröbner basis, the fundamental trilogy, Buchberger's algorithm

5. *Integer Programming with Test Sets*
   Elimination, ideal quotients, finite generation of toric ideals, testing feasibility

Literature

(1) *T. Becker, V. Weisspfenning, H. Kredel*, Gröbner Bases: A Computational Approach to Commutative Algebra, Springer, Berlin, 1993.

(2) *A. M. Bigatti, R. La Scala, L. Robbiano*, Computing toric ideals, J. Symbolic Computation **27** (1999), 351-365.

(3) *P. Conti, C. Traverso*, Buchberger Algorithm and Integer Programming, Proceedings AAECC-9, Springer, LNCS **539**, 130-139, (1991).

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(6) *R. Hemmecke*, On the decomposition of test sets: building blocks, connection sets, and algorithms, Dissertation, Gerhard-Mercator-Universität Duisburg, 2001.

(7) *Bernd Sturmfels*, Gröbner Bases and Convex Polytopes, Springer, Berlin, 1995.

(8) *R. R. Thomas*, A geometric Buchberger algorithm for integer programming, Mathematics of Operations Research **20** (1995), 864-884.

(9) *R. R. Thomas*, Gröbner Bases in Integer Programming, Handbook of Combinatorial Optimization (Eds.: D.-Z. Du and P.M. Pardalos) (1998), 533-572.

(10) *R. Urbaniak, R. Weismantel, G. M: Ziegler*, A variant of the Buchberger algorithm for integer programming, SIAM Jornal Discrete Math. **10** (1997), 96-108.

(11) *R. Hemmecke*, www.4ti2.de, under this link one can find tools for calculating test sets.