

# Nearrings and Designs

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## 1 Introduction

Motivation: Endomorphisms of groups and arbitrary selfmaps on groups. The full transformation nearring  $(M(G), +, \circ)$  on a group  $G$  and its subnearrings. Nearrings. Nearfields.

## 2 99 Years of Nearrings and Nearfields

Some historical remarks.

## 3 Related Algebraic Structures

Nearrings, nearmodules and nearalgebras and their relation to rings, modules and algebras.

## 4 Special Elements of a Nearring

Constant, zero-symmetric, distributive and antidistributive elements.

## 5 Endomorphism Nearrings and Distributively Generated Nearrings

The full endomorphism nearring  $(E(G), +, \circ)$  on a group  $G$ . Endomorphism nearrings on  $G$ . The automorphism nearring  $(A(G), +, \circ)$  and the inner auto-

morphism nearring  $(I(G), +, \circ)$  on  $G$ . Conditions on distributively generated nearrings to be a ring.

## **6 Endomorphism Nearrings on Nonabelian Groups which are Rings**

E-groups resp. A-groups resp. I-groups (i.e. groups  $G$  such that  $E(G)$  resp.  $A(G)$  resp.  $I(G)$  is a ring). Group equations characterizing I-groups. I-groups without elements of order 3. Groups of exponent 3. Free Burnside groups of exponent 3.

## **7 Endomorphism Nearrings on Finite Permutation Groups and on Linear Groups**

Coincidences in the chain  $I(G) \subseteq A(G) \subseteq E(G) \subseteq M_0(G)$  for finite permutation groups and finite linear groups.

## **8 Applications of Nearrings**

An outlook on nearrings and designs.