Lecture 2:

The Traveling Salesman Problem: Basics, Applications and Variations

TSP: original application (folklore)

- We are given a set of n cities and an $n \times n$ matrix c denoting the distance among cities.
- A traveling salesman is required to visit each city exactly once by making a unique tour whose length is a minimum.
- When the matrix c is symmetric, we have the Symmetric TSP, while in the general case we have the Asymmetric TSP.

STSP: Model (1)

Given an undirected graph $G=(V,E)$, $|V| = n$,

for each edge $e \in E$, let x_e be a binary variable taking the value 1 if e is in the tour, 0 otherwise.

$$\min \sum_{e \in E} c_e x_e$$

$$\sum_{e \in \delta(i)} x_e = 2 \quad \forall i \in V$$

$$\sum_{e \in \delta(S)} x_e \geq 2 \quad \forall S \subset V$$

$$x \in \{0;1\}^{|E|}$$

STSP: Model (2)

$$\min \sum_{(i;j) \in E} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 2 \quad \forall i \in V$$

$$u_i - u_j + (n-1) x_{ij} \le (n-2) \quad 2 \le i;j \le n$$

$$x \in \{0;1\}^{|E|}$$

Model (1) vs Model (2)

- Model (1) has an exponential number of constraints, the so-called subtour elimination constraints, but they can be separated in polynomial time and not all of them are necessary.
- The approximation given by solving Model (1) without the integrality requirement is much tighter than the one of Model (2).

TSP heuristics

- The history of Combinatorial Optimization cannot prevent from heuristics for the TSP.
- Such heuristics can be partitioned in two main classes (with many kinds of hybridizations):
  - **Tour construction heuristics**
  - **Local search methods**

Christofides heuristic (1)

- It is the most classical tour construction heuristic since 1976 and still the one providing the best performance guarantee 3/2 (provided the triangle inequality holds).

- The algorithm is particularly elegant and makes use of classical graph theory results.

Christofides heuristic (2)

- Outline:
  1. compute a minimum spanning tree;
  2. add a minimum-weight matching on the odd-degree vertices of the spanning tree;
  3. find an Eulerian tour and traverse it, shortcutting past previously visited vertices.

Lin-Kernigan heuristic (1)

- This heuristic dating 1973 is probably one of the milestones of Combinatorial Optimization.
- Main idea:
  - Starting from a feasible initial tour *T*, such a tour is iteratively improved by exploring a "neighborhood", i.e., a set of tours which can be obtained from *T* through local modifications, called "moves".

Lin-Kernigan heuristic (2)

- A classical kind of move is, e.g., a *k*-opt:
  - *k* edges of the starting tour are replaced and the best replacement is then selected.
- The move in the original paper can be interpreted as a 3-opt followed by a clever sequence of 2-opt moves, the last of which leading to an improvement in the tour length. <u>Implementation details are crucial!</u>

Classical TSP applications

- Machine scheduling problems
- Cellular Manufacturing
- Arc Routing Problems
- Frequency Assignment Problem
- X-Ray crystallography

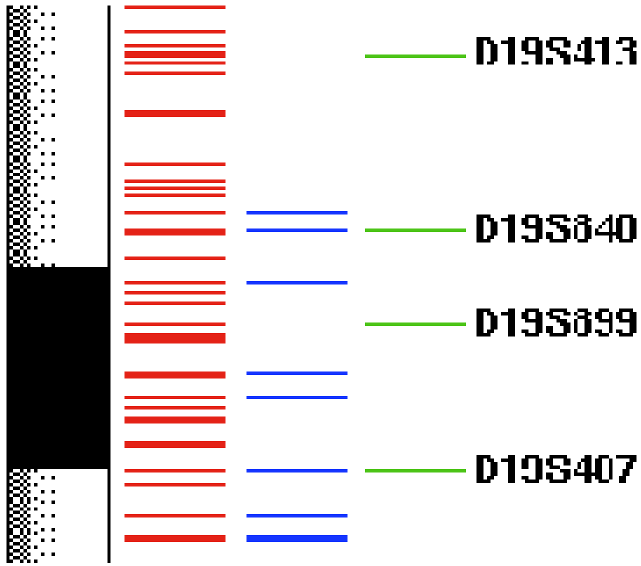
The Concorde code and recent TSP applications

- Since the early 90s, Applegate, Bixby, Chvátal, Cook developed a sophisticated branch-and-cut method to solve STSP of very large size, up to 50,000 cities:

  http://www.tsp.gatech.edu/index.html
- Their code, called “Concorde”, was used to solve a bunch of new interesting TSP applications.

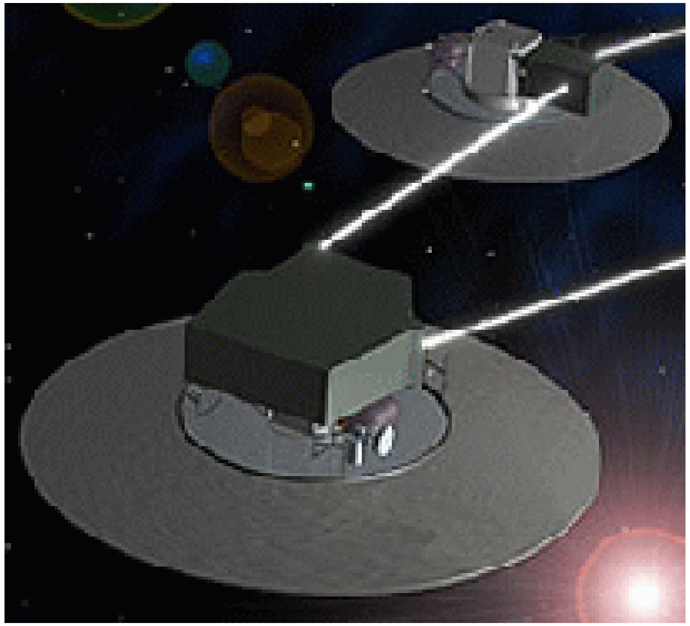
Genome sequencing

- Researchers at the National Institute of Health have used Concorde's TSP solver to construct radiation hybrid maps as part of their ongoing work in genome sequencing. The TSP provides a way to integrate local maps into a single radiation hybrid map for a genome; the cities are the local maps and the cost of travel is a measure of the likelihood that one local map immediately follows another.



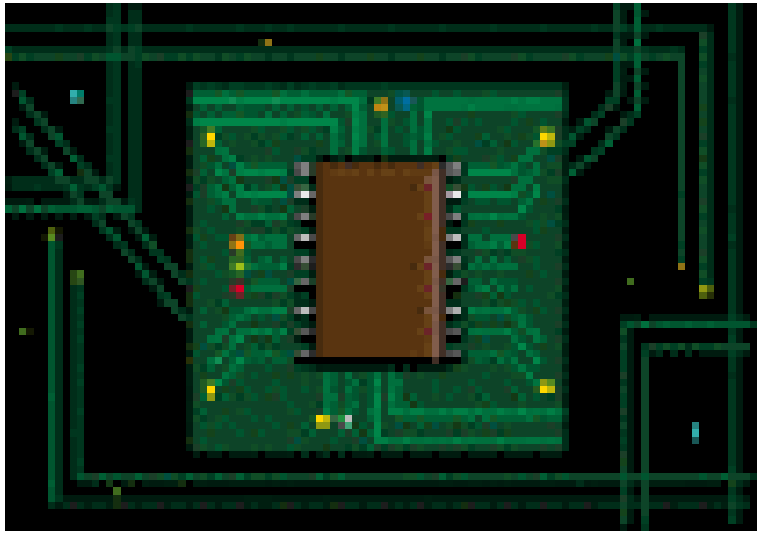
Starlight Interferometer Program

- A team of engineers at Hernandez Engineering, Houston and at Brigham Young University have experimented with using Chained Lin-Kerninghan to optimize the sequence of celestial objects to be imaged in a proposed NASA *Starlight* space interferometer program. The goal of the study is to minimize the use of fuel in targeting and imaging maneuvers for the pair of satellites involved in the mission (the cities in the TSP are the celestial objects to be imaged, and the cost of travel is the amount of fuel needed to reposition the two satellites from one image to the next).



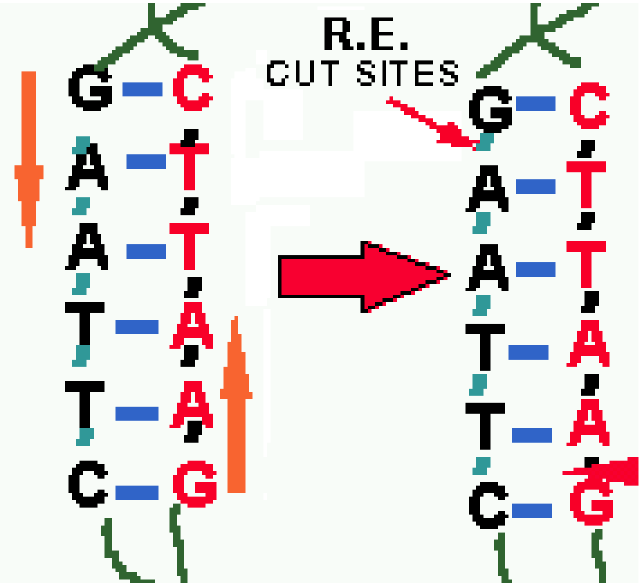
Scan Chain Optimization

- A semi-conductor manufacturer has used Concorde's implementation of the Chained Lin-Kernighan heuristic in experiments to optimize scan chains in integrated circuits. Scan chains are routes included on a chip for testing purposes and it is useful to minimize their length for both timing and power reasons.



DNA Universal Strings

- A group at AT&T used Concorde to compute DNA sequences in a genetic engineering research project. In the application, a collection of DNA strings, each of length *k*, were embedded in one universal string (that is, each of the target strings is contained as a substring in the universal string), with the goal of minimizing the length of the universal string. The cities of the TSP are the target strings, and the cost of travel is *k* minus the maximum overlap of the corresponding strings.

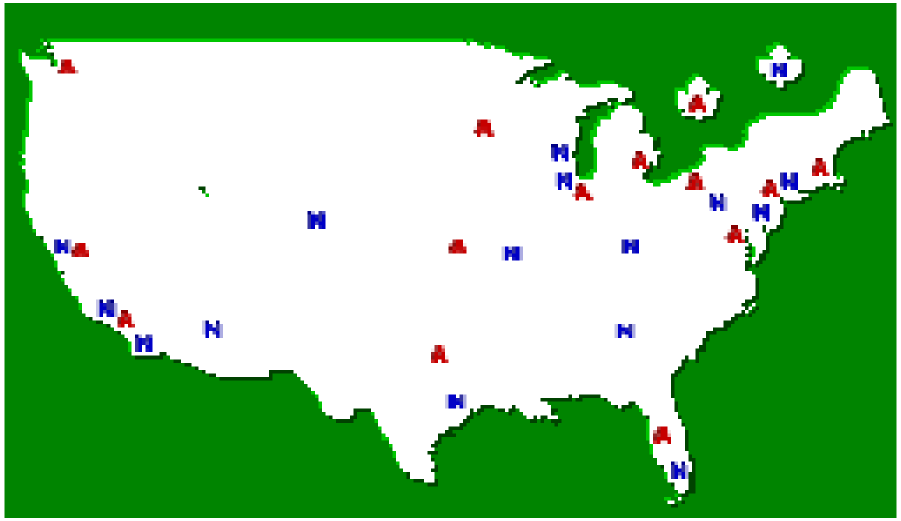


Whizzkids '96 Vehicle Routing

- A modified version of Concorde was used to solve the Whizzkids'96 vehicle routing problem, demonstrating that the winning solution in the 1996 competition was in fact optimal. The problem consists of finding the best collection of routes for 4 newsboys to deliver papers to their 120 customers. The team of David Applegate, William Cook, Sanjeeb Dash, and Andre Rohe received a 5,000 Gulden prize for their solution in February 2001 from the information technology firm CMG.

A Tour Through MLB Ballparks

- A baseball fan found the optimal route to visit all 30 Major League Baseball parks using Concorde's solver. The data for the problem can be found in the file mlb30.tsp (the data set is in TSPLIB format).



Collecting Coins

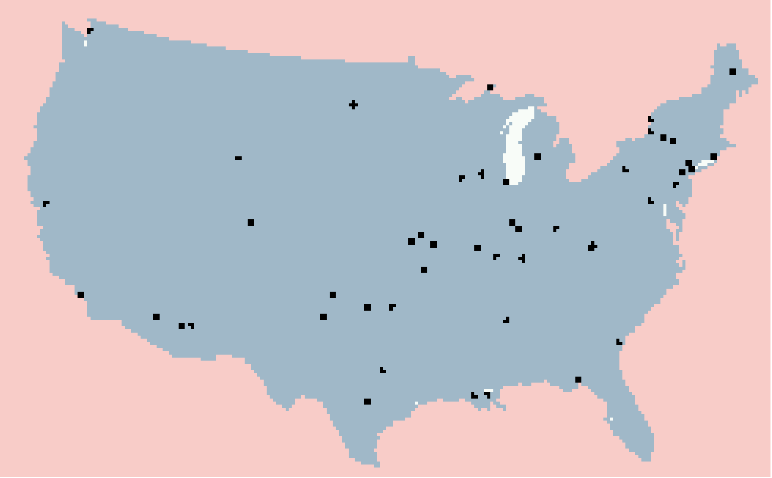
- An old application of the TSP is to schedule the collection of coins from payphones throughout a given region. A modified version of Concorde's Chained Lin-Kernighan heuristic was used to solve a variety of coin collection problems. The modifications were needed to handle 1-way streets and other features of city-travel that make the assumption that the cost of travel from ***x*** to ***y*** is the same as from ***y*** to ***x*** unrealistic in this scenario.

Touring Airports

- Concorde is currently being incorporated into the Worldwide Airport Path Finder web site to find shortest routes through selections of airports in the world. The author of the site writes that users of the path-finding tools are equally split between real pilots and those using flight simulators.

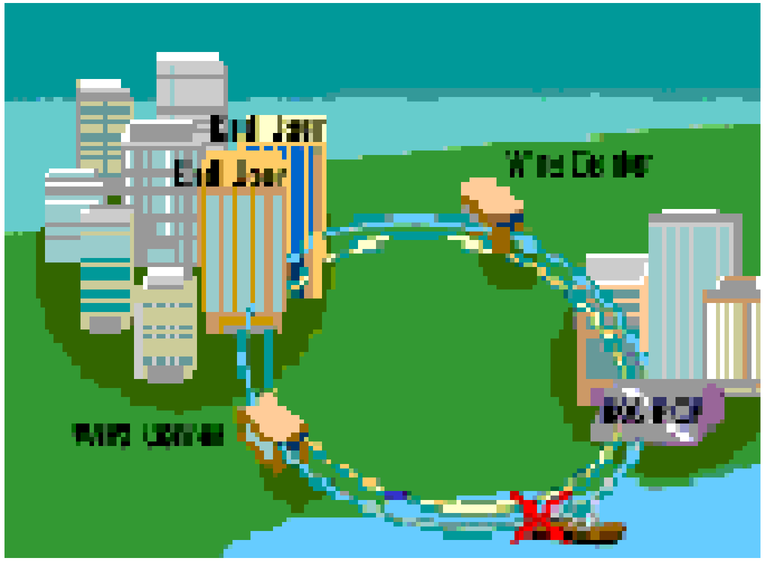
USA Trip

- The travel itinerary for an executive of a non-profit organization was computed using Concorde's TSP solver. The trip involved a chartered aircraft to visit cities in the 48 continental states plus Washington, D.C. (Commercial flights were used to visit Alaska and Hawaii.) The data for the instance was collected by Peter Winker of Lucent Bell Laboratories.



Designing Sonet Rings

- An early version of Concorde's tour finding procedures was used in a tool for designing fiber optical networks at Bell Communications Research (now Telcordia). The TSP aspect of the problem arises in the routing of sonet rings, which provide communications links through a set of sites organized in a ring. The ring structure provides a backup mechanism in case of a link failure, since traffic can be rerouted in the opposite direction on the ring.



Power Cables

- Modules from Concorde were used to locate cables to deliver power to electronic devices associated with fiber optic connections to homes.

TSP simple variations (1)

- A not exhaustive list of TSP (simple) variants includes:
  - **The MAX TSP**: the objective is to find a tour where the total cost is a maximum.
  - **The bottleneck TSP**: the objective is to find a tour such that the largest cost of an edge in the tour is as small as possible.
  - **TSP with multiple visits**: the objective is to find the shortest tour but one is allowed to visit each city more than once.

TSP simple variations (2)

- **Messenger problem**: the objective is to find a least-cost Hamiltonian path between two specified nodes *u* and *v*.

- **Clustered TSP**: V is partitioned into clusters $V_1,\ldots, V_k$ and the objective is to find the shortest tour such that vertices in the same cluster must be visited consecutively.

TSP simple variations (3)

– **Generalized TSP**: V is again partitioned into clusters $V_1, \ldots, V_k$ and the objective is to find a cycle such that exactly one node in each cluster is visited, i.e., visiting k cities.

– **The *m*-salesmen TSP**: we are given m salesmen, located at a specific node, say 1, and the objective is to find m disjoint cycles, starting and returning at node 1, whose overall length is minimized.

More TSP variants

- Time dependent TSP
- Period TSP
- Delivery man Problem
- Black & White TSP
- Angle TSP
- Film-copy Deliver
- *k*-Peripatetic Salesman Problem
- Covering Salesman Problem
- Selective TSP
- Resource constrained TSP
- Serdyukov TSP
- Ordered Cluster TSP
- Precedence constrained TSP
- TSP with Time Windows
- Moving Target TSP
- Remote TSP