Quantum Mechanics
July, 6 2017

PROBLEM A

A two level system is described by the Hamiltonian

$$H = \hbar\omega \begin{pmatrix} 4 & 3i \\ -3i & -4 \end{pmatrix}. \tag{1}$$

Initially is in the state $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

1. Determine eigenvalues and eigenvectors of H.
2. What is the probability to found the system at the time t in the state $|2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$?
3. Determine at what time t the probability to find the system in state $|2\rangle$ is $1/2$.

PROBLEM B

Consider a particle with mass m and charge q in a 3d harmonic potential

$$V(x, y, z) = \frac{1}{2}k(x^2 + y^2 + z^2) \equiv \frac{1}{2}k|\vec{r}|^2$$

Switch on a uniform magnetic field B along z. Neglecting the spin degrees of freedom the Hamiltonian is given by,

$$H = \frac{1}{2m}(\vec{p})^2 + \frac{1}{2}k|\vec{r}|^2 - \frac{qB}{2mc}L_z.$$

1. Re-write the Hamiltonian using creation and annihilation operators for the harmonic oscillator.
2. Using perturbation theory for small B find the smallest corrections for the first two energy levels.