Continuum shell model: From Ericson to conductance fluctuations

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We discuss an approach for studying the properties of mesoscopic systems, where discrete and continuum parts of the spectrum are equally important. The approach can be applied (i) to stable heavy nuclei and complex atoms near the continuum threshold, (ii) to nuclei far from the region of nuclear stability, both of the regions being of great current interest, and (iii) to mesoscopic devices with interacting electrons. The goal is to develop a new consistent version of the continuum shell model that simultaneously takes into account strong interaction between fermions and coupling to the continuum. Main attention is paid to the formation of compound resonances, their statistical properties, and correlations of the cross sections. We study the Ericson fluctuations of overlapping resonances and show that the continuum shell model nicely describes universal properties of the conductance fluctuations.

A. Introduction

The focus of interest in low-energy nuclear physics has recently concentrated on nuclei far from the region of stability. The success of the nuclear shell model with strong interaction between nucleons urges one to look for ways of incorporating this rich experience into a more general context that would properly include the continuum. The continuum shell model is expected to combine the physics of intrinsic structure and reactions in contrast to the traditional spectroscopy and reaction theory which were historically developed as different branches of nuclear physics. In this approach, the properties of open channels, threshold behavior, symmetries, and spectroscopic factors enter the overall theory, so that a fully consistent picture uniting structure and reaction mechanisms is created. With great successes of mesoscopic physics (complex atoms and molecules, micro- and nano-devices of condensed matter, atomic and molecular traps, and prototypes of quantum computers), the nuclear problem becomes a typical part of general science of open mesoscopic systems.

In this paper we discuss a new approach [1] for studying collective phenomena in nuclei and quantum dots which emerge as a result of strong interactions between fermions coupled to the continuum. In this problem the standard shell model techniques, adjusted for the bound states, cannot work and continuum effects must be properly accounted for. At a certain strength of the continuum coupling, the system of strongly interacting fermions undergoes a collective restructuring. This is an area where the conventional division of nuclear physics into "structure" and "reactions" becomes inappropriate and two views of the process, from the inside (structure and properties of bound states) and from the outside (cross sections of reactions), have to be unified.

Conventional theory of statistical reactions does not answer the question of interplay between reactions and internal structure determined by the character of interactions between the constituents. Here more detailed considerations are required based on the generalization of the shell model of nuclear reactions [2]. Such an extension requires statistical assumptions concerning intrinsic dynamics and the coupling to the continuum [3, 4]. To account for specificity of the system, one has to go beyond standard random matrix approaches [5, 6, 7] utilizing the Gaussian Orthogonal Ensemble (GOE). The consistent description based on the continuum shell model [2], as well as more phenomenological approaches [8, 9], indicates the presence of a sharp restructuring of the system when the widths of resonances become comparable to their energy spacings. This phenomenon carries features of a quantum phase transition with the strength of continuum coupling playing the role of a control parameter.

As was numerically observed in the shell model framework [10], the distribution of resonance widths rapidly changes in the transitional region in such a way that a number of very broad resonances equal to a number of open decay channels absorb the lion's share of the total width of all overlapped resonances, while the remaining states become very narrow. The corresponding theory was suggested in [11, 12, 13], where the mechanism of this restructuring was understood to be associated with the nature of the effective non-Hermitian Hamiltonian [2] that describes the intrinsic dynamics after eliminating the channel variables. The segregation of short-lived broad resonances from long-lived trapped states was shown to be similar to the superradiance [14] in quantum optics induced by the coupling of atomic radiators through the common radiation field, an analog of coherent coupling of many overlapped intrinsic states through continuum decay channels. Later a general character of the phenomenon was demonstrated for systems with the GOE intrinsic dynamics and many open channels [15, 16]. Modern versions of the shell model in continuum

[17, 18] are based on the effective Hamiltonian and naturally reveal the superradiance phenomenon as an important element. The transition to this regime should be taken into account in all cases when a physical system is strongly coupled to the continuum, see for example [19] and references therein.

In the present work, using the same framework as in [1], we study the interplay between the intrinsic dynamics and statistical properties of cross sections comparing the results with those of conventional approaches, namely Hauser-Feshbach average cross sections and Ericson fluctuations and correlations. We also analyze how the universal conductance fluctuations appear in our model, how they depend on the intrinsic chaos and what is the role of correlations between cross sections of various processes.

B. The model

We consider a generic system of n interacting fermions distributed over m single-particle levels of energies ϵ_s ("mean-field orbitals"). The fermions are coupled to continuum through the finite number M of open decay channels. In the many-body representation, the Hamiltonian matrix of size $N = m!/[n!(m-n)!]$ entirely determines the intrinsic dynamics of the closed system, and our main interest is to reveal how the properties of the reactions depend on the degree of intrinsic chaos that is due to the inter-particle interaction. According to the well developed formalism [2, 12, 19], the behavior of an open system can be described in terms of the effective non-Hermitian Hamiltonian consisting of two terms,

$$\mathcal{H} = H - \frac{i}{2} W\,; \qquad W_{ij} = \sum_{c=1}^{M} A_i^c A_j^c. \tag{1}$$

Here H is a Hermitian matrix that can be also influenced by the presence of the continuum [17, 18, 20], and W stands for the coupling between intrinsic many-body states labeled as $i, j, ...$, and open decay channels $a, b, c....$ The factorized structure of W is dictated by the requirement of the unitarity of the scattering matrix. In what follows we assume that the transition amplitudes A_i^c between intrinsic states $|i\rangle$ and channels c are real quantities, therefore, we restrict ourself by the time-invariant systems. As a result, both H and W are real symmetric matrices.

The Hermitian part, $H = H_0 + V$, of the full Hamiltonian (1) is modeled by the so-called two-body random interaction (TBRI) (see, for example, [21] and references therein),

$$H = H_0 + V; \qquad H_0 = \sum_{s=1}^{m} \epsilon_s \, a_s^\dagger a_s; \qquad V = \frac{1}{2} \sum V_{s_1 s_2 s_3 s_4} \, a_{s_1}^\dagger a_{s_2}^\dagger a_{s_3} a_{s_4}. \tag{2}$$

Here the mean-field part H_0 describes non-interacting particles (or quasi-particles), and the interaction between particles is given by V. In the many-particle basis $|k\rangle$ the matrix H is constructed by the Slater determinant, $|k\rangle = a_{s_1}^\dagger \ldots a_{s_n}^\dagger |0\rangle$, where $a_s^\dagger$ and a_s are the creation and annihilation operators. The interaction between the particles is considered to be of two-body nature, therefore, each many-body matrix element $V_{lk} = \langle l|V|k\rangle$ is a sum of a number of two-body matrix elements $V_{s_1 s_2 s_3 s_4}$ involving at most four single-particle states $|s\rangle$. In our simulations we take $n = 6$, $m = 12$ that provides a sufficiently large dimension $N = 924$.

The *single-particle* energies, ϵ_s, are assumed to have a Poissonian distribution of spacings, with the mean level density $1/d_0$. The interaction V is characterized by the variance of the normally distributed two-body random matrix elements, $\langle V_{s_1,s_2;s_3,s_4}^2 \rangle = v_0^2$. Without the interaction, $v_0 = 0$, the many-body states have also the Poissonian spacing distribution $P(s)$. In the opposite limit of an extremely strong interaction, $\lambda \equiv v_0/d_0 \to \infty$ (or, equivalently, for $d_0 = 0$), the function $P(s)$ is close to the Wigner-Dyson (WD) distribution that is typical for completely chaotic systems [5]. The critical value of the interaction for the onset of strong chaos was obtained in Ref. [21], it is estimated as

$$\lambda_{\rm cr} = \frac{v_{\rm cr}}{d_0} \approx \frac{2(m-n)}{N_s}, \tag{3}$$

where $N_s = n(m-n) + n(n-1)(m-n)(m-n-1)/4$ is the number of directly coupled many-body states in any row of the matrix H_{ij}. One can expect that for a very strong interaction, $\lambda \gg \lambda_{\rm cr}$, some of properties of the system may be close to those obtained with H taken from the Gaussian Orthogonal Ensemble (GOE). For this reason, we have also considered, for comparison, the GOE matrices in place of H.

In our study the amplitudes A_i^c are assumed to be random independent Gaussian variables with the zero mean and variance

$$\langle A_i^c A_j^{c'} \rangle = \delta_{ij} \delta^{cc'} \frac{\gamma^c}{N}. \tag{4}$$

This is compatible with the GOE or TBRI models where generic many-body states coupled to continuum have a very complicated structure, while the decay probes specific simple components of these states related to few open channels. Even for a weak intrinsic interaction, one needs to have in mind that in reality the states $|i\rangle$ have certain values of exact constants of motion, such as angular momentum and isospin in the nuclear case. Therefore, at sufficiently large dimension N, these states acquire *geometric chaoticity* [22] due to the almost random coupling of individual spins. Thus, the assumption of a random nature of the decay amplitudes is reasonable.

The parameters γ^c characterize the total coupling of all states to the channel c. The normalization used in Eq. (4) is convenient if the energy interval ND covered by decaying states is finite. Here D is the distance between the many-body states in the middle of the spectrum, $D = 1/\rho(0)$, where $\rho(E)$ is the many-body level density, and $E = 0$ corresponds to the center of the spectrum. Below we neglect a possible energy dependence of the amplitudes that is important near thresholds and is taken into account in realistic shell model calculations [18, 20]. The ratio γ^c/ND characterizes the degree of overlap of the resonances in the channel c. The control parameter determining the strength of the coupling to the continuum can be written as follows,

$$\kappa^c = \frac{\pi\gamma^c}{2ND}. \tag{5}$$

The transition from separated to strongly overlapped resonances corresponds to $\kappa^c \approx 1$. In order to keep the coupling to continuum fixed, in our numerical calculations we renormalize the absolute magnitude of the widths, γ^c, by varying the intrinsic interaction and, therefore, the level density ρ.

The effective Hamiltonian allows one to construct the scattering matrix,

$$S^{ab} = \delta^{ab} - i\mathcal{T}^{ab}, \tag{6}$$

where

$$\mathcal{T}^{ab}(E) = \sum_{i,j}^{N} A_i^a \left(\frac{1}{E - \mathcal{H}}\right)_{ij} A_j^b \tag{7}$$

are the scattering amplitudes determining the cross sections $\sigma^{ab}(E)$ of reactions,

$$\sigma^{ab}(E) = |\mathcal{T}^{ab}(E)|^2. \tag{8}$$

In our notations the cross sections are dimensionless since we omit the common factor π/k^2. In what follows we study both the elastic, $b = a$, and inelastic, $b \neq a$, cross sections. Note that we ignore the smooth potential phases that are irrelevant for our purposes.

C. Transition to superradiance

The complex eigenvalues $\mathcal{E}_r = \omega_r - (i/2)\Gamma_r$ of $\mathcal{H}$ coincide with the poles of the S-matrix and, for small γ^c, determine energies and widths of separated resonances. In the simulations we consider a real energy interval at the center of the spectrum of $\mathcal{H}$ with the constant many-body level density $\rho(0) = D^{-1}$. The transmission coefficient in the channel c,

$$T^c = 1 - |\langle S^{cc}\rangle|^2 = \frac{4\kappa^c}{(1+\kappa^c)^2}, \tag{9}$$

is maximal (equal to 1) at the critical point, $\kappa^c = 1$, and, for simplicity, we assume M equiprobable channels, $\kappa^c = \kappa$. For each value of κ we have used a large number of realizations of the Hamiltonian matrices, with further averaging over energy.

As γ^c grows, the resonances start to overlap, leading to a segregation of the resonance widths [10, 11, 12, 23] occurring at $\kappa \approx 1$, see Fig. 1. The widths of M resonances are increasing at the expense of the remaining $N - M$ resonances. For a weak coupling, $\kappa \ll 1$, the widths are given by diagonal matrix elements, $\Gamma_i = \langle i|W|i\rangle = \sum_{c=1}^{M}(A_i^c)^2$, and the mean width is $\langle\Gamma\rangle = \gamma M/N$. In the limit of strong coupling, $\kappa \gg 1$, the widths of M broad resonances converge to the non-zero eigenvalues of the matrix W that has a rank M due to its factorized structure. As for the remaining "trapped" $(N - M)$ states, their widths decrease as $1/\gamma$. From our data one can conclude that the transition to the superradiance in dependence on the control parameter κ turns out to be quite sharp, and may be treated as a kind of the phase transition.

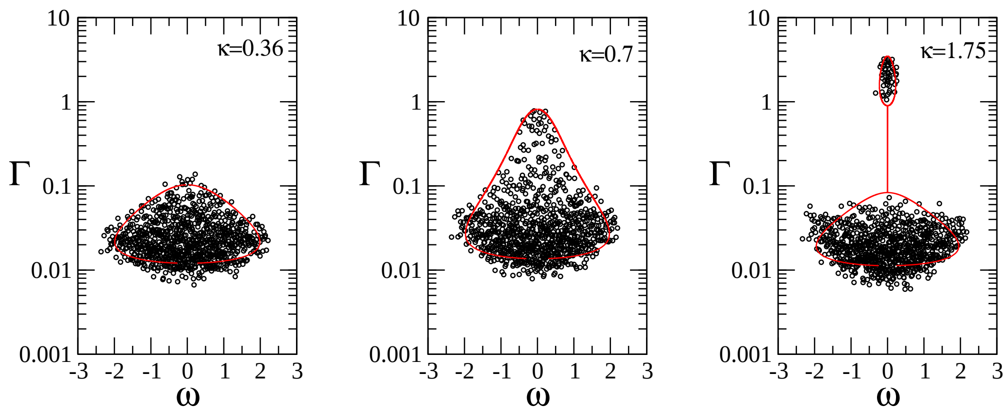

FIG. 1: Distribution of widths, Γ, and positions, ω, of resonances (poles of the scattering matrix) as a function of κ. The results are shown for $n = 6$ particles, $m = 12$ orbitals, 50 open channels, and for a relatively strong interaction between particles, $v_0/d_0 = 1/10$, above the strong chaos threshold. The smooth curves are the boundaries due to the analytical results obtained in Ref.[23] for the GOE matrices in place of H

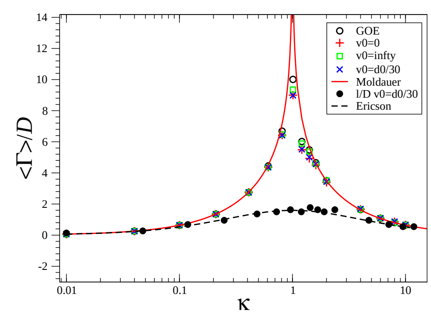

FIG. 2: Normalized average width versus κ for $n = 6, m = 12$ and $M = 10$. The average was performed over the resonances around the center of the energy spectrum. Open circles refer to the GOE case, pluses to $\lambda = 0$, squares to $\lambda \to \infty$, and crosses to $\lambda = 1/30$. Solid curve shows the MS-expression (10). Full circles stand for the normalized correlation length at $\lambda = 1/30$, see in the text. The dashed curve shows the Weisskopf relation (16).

In Fig. 2 we show how the *average width* (normalized to the mean level spacing D) depends on κ for different values of the interaction λ. The figure demonstrates a sharp change in the width distribution at the transition point $\kappa = 1$. The results are compared with the expression

$$\frac{\langle \Gamma \rangle}{D} = \frac{M}{\pi} \ln \left| \frac{1+\kappa}{1-\kappa} \right| \tag{10}$$

that is obtained from the Moldauer-Simonius (MS) relation [8, 24] for M equivalent channels. We have extrapolated the MS-relation to the region with $\kappa > 1$ for which only $N - M$ narrow resonances were taken into account. For one channel in the GOE case this result was obtained in Refs. [11, 12, 15]; here we see that the MS-expression is also valid for a large number of channels. Note that the results are independent of the degree of intrinsic chaoticity; this fact was explained in Ref.[1]. According to [16], the divergence of $\langle \Gamma \rangle$ at $\kappa = 1$ is due to the (non-integrable) power-law behavior for large Γ, see below; in the numerical simulation, see Fig. 2, there is no divergence because of the finite number of resonances, although the trend is clearly seen.

As for the width distribution $P(\Gamma)$, in Ref. [1] it was shown that for a weak coupling, $\kappa \ll 1$, the conventional χ^2_M-distribution is valid for any strength of the interaction. However, when κ grows, a clear dependence on the interaction strength emerges. As first noted in Ref. [25], as κ increases, $P(\Gamma)$ becomes broader than the χ^2_M distribution. For large κ, both for the GOE and for $\lambda \to \infty$, $P(\Gamma)$ is again given by the χ^2_M distribution, contrary to the cases of the finite interaction strength. Our results are in a good agreement with the analytical predictions [16, 26] obtained for the GOE case (neglecting the emergence of broad resonances in the regime of strong overlap at $\kappa > 1$). In particular, for large number of channels, the tails of $P(\Gamma)$ decrease as Γ^{-2}, thus leading to the divergence of $\langle \Gamma \rangle$ at $\kappa = 1$.

D. Ericson fluctuations

According to the conventional theory of Ericson fluctuations [27, 28, 29], the scattering amplitude can be written as the sum of the average and fluctuating parts, $\mathcal{T}^{ab}(E) = \langle \mathcal{T}^{ab}(E)\rangle + \mathcal{T}^{ab}_{\rm fl}(E)$, with $\langle \mathcal{T}^{ab}_{\rm fl}(E)\rangle = 0$. Correspondingly, the average cross section, $\sigma = |\mathcal{T}|^2$, can be also divided into two contributions, $\langle\sigma\rangle = \langle\sigma_{\rm dir}\rangle + \langle\sigma_{\rm fl}\rangle$. Here the direct reaction cross section, $\langle\sigma_{\rm dir}\rangle$, is determined by the average scattering amplitude only, while $\langle\sigma_{\rm fl}\rangle$ is the fluctuational cross section (also known as the compound nucleus cross section).

The main assumption of the standard theory is that in the regime of overlapping resonances, $\langle\Gamma\rangle > D$, the fluctuating amplitudes can be written as $\mathcal{T}^{ab}_{\rm fl}(E) = \xi + i\eta$ where both ξ and η are Gaussian random variables with zero mean. This is supported by the fact that for $\langle\Gamma\rangle \gg D$ both ξ and η are the sums of a large number of random variables. This assumption can be associated with the other one, namely, with statistical independence of poles (resonance energies) and residues (resonance amplitudes). As a result, the statistical methods can be developed that lead to various conclusions. In the frame of our model we have tested the most important of the Ericson predictions concerning the resonance widths and cross sections (see also Refs.[1]), paying special attention to the dependence on the intrinsic interaction strength, λ.

a. Fluctuations of resonance widths In the theory of Ericson fluctuations it is usually assumed that the fluctuations of resonance widths are small for a large number of channels, $w(\Gamma)/\langle\Gamma\rangle^2 \ll 1$, where $w(\Gamma)$ stands for the variance of widths. In justification of this statement [27] it is argued that in the overlapping regime the width of a resonance can be presented as a sum of M partial widths. Assuming that they obey the Porter-Thomas distribution, the total width is expected to have a χ^2_M distribution, so that $w(\Gamma)/\langle\Gamma\rangle^2 = 2/M$ is small for $M \gg 1$.

However, recently we have found [1] that for large values of κ the distribution of the widths strongly differs from the χ^2_M distribution. The data show that, as κ increases, the normalized variance, $w(\Gamma)/\langle\Gamma\rangle^2$, also increases, remaining very large even for $M = 20 \gg 1$. Thus, contrary to the traditional belief, the relative variance of widths does not become small for large number of channels. The data confirm that for non-overlapping resonances (small $\kappa \ll 1$) there is no dependence on λ and $w(\Gamma)/\langle\Gamma\rangle^2$ decreases as $2/M$ for all the ensembles, as expected. However, as κ grows, the dependence on λ emerges: the weaker the intrinsic chaos (and, consequently, the more ordered is the intrinsic spectrum) the larger are the width fluctuations. For large $\kappa \gg 1$ and strong interaction between particles, $\lambda > \lambda_{cr}$, the width distribution is again given by the χ^2_M distribution, contrary to the cases of the finite interaction strength.

b. Cross sections Speaking about cross sections and their fluctuations, it is convenient to use the so-called *elastic enhancement factor*, that for M equivalent channels takes the form,

$$F = \frac{\langle\sigma^{aa}_{\rm fl}\rangle}{\langle\sigma^{ab}_{\rm fl}\rangle}, \tag{11}$$

where $b \neq a$. Here $\langle\sigma^{ab}_{\rm fl}\rangle$ stands for the fluctuational part of cross sections (elastic for $a = b$ and inelastic for $a \neq b$). In the case of equal channels, we obtain

$$\langle\sigma^{ab}_{\rm fl}\rangle = \frac{1 - |\langle S^{aa}\rangle|^2}{F + M - 1} = \frac{T}{F + M - 1}, \tag{12}$$

therefore, $\langle\sigma^{aa}_{\rm fl}\rangle = F\langle\sigma^{ab}_{\rm fl}\rangle$, where T is the transmission coefficient, see Eq. (9). Since the transmission coefficient T does not depend on λ, the only dependence on λ in Eq. (12) is contained in the elastic enhancement factor F. The same seems to be correct even when the channels are non-equivalent, according to the results of Ref. [30]. It should be pointed out that F also depends on κ. Specifically, with an increase of κ from zero, the value of F decreases, being confined by the interval between 3 and 2, see discussion in Ref. [1].

As one can see from Eq.(12), with an increase of the number of channels the dependence on the interaction strength λ disappears for the fluctuational *inelastic* cross section. In contrast, the fluctuational *elastic* cross section manifests a clear dependence on λ, according to the data of Ref. [1]. This fact allows one to directly relate the value of the enhancement factor F to the strength λ of interaction between the particles. Specifically, the more regular is the intrinsic motion, the higher is the average cross section. For a large number of channels, the λ-dependence of the elastic cross section is in agreement with the estimate, $\langle\sigma^{aa}_{\rm fl}\rangle \to FT/M$.

c. Fluctuations of cross sections The fluctuations of both elastic and inelastic cross sections strongly depend on the coupling to the continuum, see examples in Fig. 3. According to the standard Ericson theory, in the region of strongly overlapping resonances, $\kappa \approx 1$, the variance of fluctuations of both elastic and inelastic cross sections can be expressed via the average cross sections. Specifically, for equal channels one gets,

$$\mathrm{Var}(\sigma) = \langle\sigma_{\rm fl}\rangle\,(2\langle\sigma_{\rm dir}\rangle + \langle\sigma_{\rm fl}\rangle) = \langle\sigma_{\rm fl}\rangle^2. \tag{13}$$

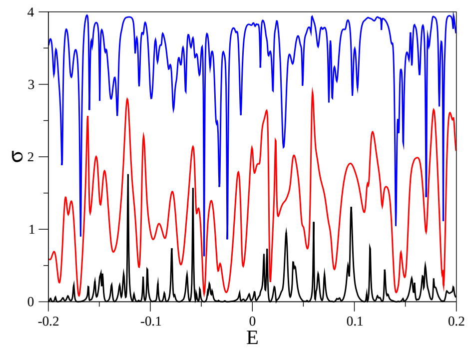

FIG. 3: Elastic cross-section versus energy E for different strength of coupling to the continuum: for weak, $\kappa = 0.07$, strong, $\kappa = 0.7$, and very strong, $\kappa = 7.0$, for $M = 10$. Other parameters are the same as in Fig.1.

The latter equality is written for the absence of direct processes, that is our case. Thus, one can expect the relation, $\mathrm{Var}(\sigma)/\langle\sigma_{\rm fl}\rangle^2 = 1$, for a large number of channels. Our data confirm this relation for inelastic cross sections, independently of the interaction strength λ, and for $M > 10$.

As for the elastic cross sections, the expression (13) emerges for a quite large number of channels, $M \geq 25$. This fact of slow convergence to the limiting value was also found analytically in Refs. [31, 32] for the GOE case, any number of channels and any coupling strength with the continuum. However, simple expressions were derived only for $MT \gg 1$. Our data reveal a clear dependence on λ that is stronger for the non-normalized variance $\mathrm{Var}(\sigma)$ than for the normalized variance $\mathrm{Var}(\sigma)/\langle\sigma_{\rm fl}\rangle^2$. This effect was explained in Ref. [1].

Another well known prediction of Ericson theory is that for strongly overlapping resonances both inelastic and elastic cross sections have the exponential distribution,

$$P(x) = e^{-x}, \quad x = \frac{\sigma_{\rm fl}}{\langle\sigma_{\rm fl}\rangle}. \tag{14}$$

Indeed, our data for inelastic cross sections show good agreement with this prediction, independently of the interaction strength λ and for large number of channels, $M \geq 10$. For the elastic cross sections, the exponential distribution arises for larger number of channels, $M \geq 25$, with a weak dependence on λ. In general, one can say that the exponential form of the fluctuations is a good approximation in the region $\kappa \approx 1$.

d. Correlation functions The correlation function of cross sections is defined as

$$C(\epsilon) = \langle\sigma(E)\sigma(E+\epsilon)\rangle - \langle\sigma(E)\rangle^2. \tag{15}$$

By assuming the Gaussian form of distribution for fluctuating amplitudes $\mathcal{T}_{\rm fl}$, one can obtain that the correlation function has a Lorentzian form, with the correlation length equal to the average width, $l_\sigma = \langle\Gamma\rangle$. In our model we have found that $l_\sigma \approx l_S$ for a large number of channels in the region $\kappa \approx 1$, and for any interaction strength λ. Here, l_S is the correlation length obtained from the correlation function of the scattering matrix. On the contrary, for smaller M, our data show that $l_S < l_\sigma$, and this difference grows for the weaker interaction between the particles.

For large M, the correlation functions have, indeed, the Lorentzian form for all λ. Meanwhile, for a small number of channels the correlation function is not Lorentzian, in agreement with the results of [33]. It is important to stress that for any M, the correlation length is different from the average width, apart from the region of small κ. Instead, for a large number of channels, the correlation length is determined by the Weisskopf relation, see [34] and references therein,

$$\frac{l}{D} = \frac{MT}{2\pi} = \frac{M}{2\pi}\frac{4\kappa}{(1+\kappa)^2}. \tag{16}$$

The Weisskopf relation (16) is confirmed by our data in Fig. 2, it has been also derived in Ref. [3] for small values of the ratio $m = M/N$, in the overlapping regime for the TBRI model with the infinite interaction, as well as in Ref. [4] for the GOE ensemble. The fact that the correlation length is not equal to the average width was recognized long ago, see Refs. [35] and [5].

E. Conductance fluctuations

Our model can be also used to study conductance and its fluctuations for quantum dots with interacting electrons. In such an application one can treat $M/2$ channels as the *left channels* corresponding to incoming electron waves, and other $M/2$ channels, as the *right channels* corresponding to outgoing waves. Then, one can define the conductance G in a standard way,

$$G = \sum_{a=1}^{M/2} \sum_{b=M/2+1}^{M} \sigma^{ab} \tag{17}$$

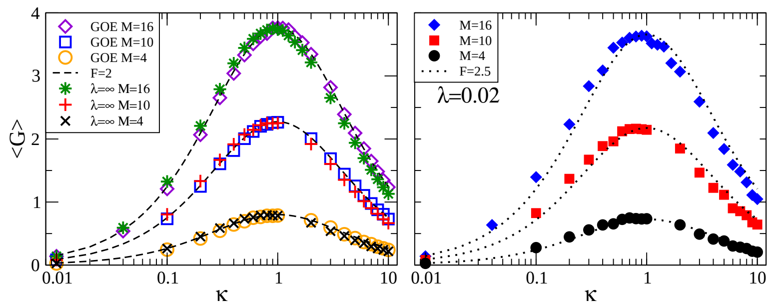

FIG. 4: Average conductance versus κ for the GOE and very strong interaction, $\lambda \to \infty$ (left panel), and for a weak interaction, $\lambda = 0.02$ (right panel) for different number of channels, $M = 4, 10, 16$. Dashed curves correspond to the analytical expression (18) with $F = 2.0$, (left panel), and dotted curves with $F = 2.5$, (right panel). The data are obtained for $n = 6$ particles, $m = 12$ orbitals, and $M = 4, 10, 16$ channels.

Therefore, the properties of the conductance are entirely determined by the inelastic cross-sections, $b \neq a$. In our case of equivalent channels the average conductance reads,

$$\langle G \rangle = \frac{M^2}{4} \langle \sigma^{ab} \rangle = \frac{M^2}{4} \frac{T}{F + M - 1} \to \frac{MT}{4} \tag{18}$$

where $T = 4\kappa/(1+\kappa)^2$ and the last expression is valid for $M \gg 1$. As one can see, the influence of the inter-particle interaction is due to the enhancement factor F only. This factor ranges from $F = 3$ for the absence of the interaction, $\lambda = 0$, to $F = 2$ for a very strong interaction that can be modeled by the GOE ensemble [36]. Since currently there is no theory relating the enhancement factor to the degree of chaos in the internal dynamics, we used this factor as the fitting parameter. In Fig. 4 we report our numerical data obtained for the average conductance in dependence on the coupling strength κ for different values of M. As expected, for a very strong interaction (absence of the mean field part, $d_0 = 0$, or, the same, $\lambda \to \infty$) the results are very close to those obtained for the GOE Hamiltonian H, and there is a good agreement with the expression (17) with the theoretical value $F = 2$. On the other hand, for a relatively weak interaction, $\lambda = 0.02$, (below the chaos threshold in a closed model) the data can be described by the same expression with $F = 2.5$ that is in the middle of the region between the Poisson and Wigner-Dyson statistics for the energy spacings. As one can see, for this intermediate case the correspondence between the relation (18) and data is also good. Note, that for a very large number of channels the influence of the internal dynamics disappears.

Our main interest was in the variance, $\mathrm{Var}(G)$, of the conductance when we change the energy E around the center of the energy spectrum. In Fig. 5 we report the data for the variance for the GOE case (left part) and for a relatively weak interaction $\lambda = 0.02$ in dependence on the coupling parameter κ and number M of channels. First observation concerns a clear evidence that for a weak coupling, $\kappa \approx 0.1$, the role of the interaction should not be neglected. This fact is in contrast with the average conductance for which the influence of the internal chaos seems to be much less in comparison with the conductance fluctuations.

Second important observation is that at the transitional point, $\kappa \approx 1$ (known in the solid state applications as "perfect coupling"), practically there is no influence of the inter-particle interaction. This fact is quite instructive, it manifests an important role of the superradiance when the (strong) interaction between the particles is mainly

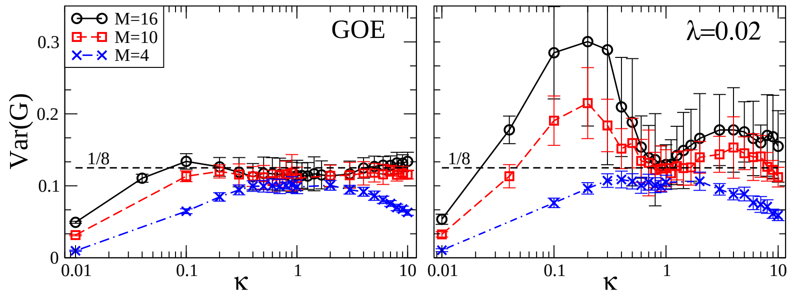

FIG. 5: Variance of the conductance versus κ for the GOE (left panel), and for $\lambda = 0.02$, (right panel) . Circles, squares and crosses stand for $M = 16, 10, 4$ channels, respectively. The data are given for $n = 6$ and $m = 12$.

due to the continuum. Also, the dependence of fluctuations on the number of channels at the transitional region is relatively weak (and disappears with an increase of M). In the theory of conductance fluctuations it is known that for a perfect coupling the variance takes the universal value in the limit of a very large number of channels. This value for time-invariant systems is either 2/15 or 1/8 depending on the type of transport (ballistic or diffusive, both for a bulk scattering, see, e.g. [37]). It is quite instructive that our data confirm the value 1/8 for the GOE at $\kappa = 1$ with a very good accuracy. Specifically, the data also reveal small corrections due to the finite values of M, that reduce the variance to that of slightly below 1/8, in agreement with the analytical results obtained in Ref. [37]). Also, the data indicate the value 2/15 for the TBRI model with a moderate interaction between the particles.

We have performed a detailed analysis of the emergence of the universal value 1/8. We should point out that one has to take into account the correlations between different cross sections in the expression for the variance of the conductance. Otherwise, one gets the incorrect result,

$$\mathrm{Var}(G) = \frac{M^2}{4}\mathrm{Var}(\sigma) = \frac{M^2}{4}\left(\frac{T}{F+M-1}\right)^2 \to \frac{T^2}{4}, \tag{19}$$

where the last expression is obtained for $M \gg 1$. Thus, for the perfect coupling, we would have $\mathrm{Var}(G) = 1/4$, which is twice the correct value.

In order to reveal the role of correlations between different cross sections, instead of Eq. (19) we write the relation that takes into account all the correlations,

$$\mathrm{Var}(G) = \frac{M^2}{4}\left(\frac{T}{F+M-1}\right)^2 + N^* C_\Sigma + (N_c - N^*) C_\Lambda, \tag{20}$$

where $N_c = L(L-1)$, $N^* = L(M-2)$, $L = M^2/4$. Here the terms C_Λ and C_Σ stand for the correlation functions,

$$C = \overline{\langle \sigma^{ab}\sigma^{a'b'}\rangle} - \overline{\langle \sigma^{ab}\rangle\langle\sigma^{a'b'}\rangle} \tag{21}$$

with one common index (either $a = a'$ or $b = b'$) for the $\Sigma-$correlations, and with no common indexes (both $a \neq a'$ and $b \neq b'$) for the $\Lambda-$correlations. The bar in Eq. (21) represents the average over different possible combinations of a, b, a', b', and $\langle ... \rangle$ indicate the average over energy. Our analysis shows that for $M \gg 1$ one obtains, $C_\Sigma \approx -M^{-3}$ and $C_\Lambda \approx 2M^{-4}$. Therefore, in the limit of large number of channels, one can write, in the correspondence with the structure of Eq.(20),

$$\mathrm{Var}(G) = \frac{1}{4} - \frac{1}{4} + \frac{1}{8} = \frac{1}{8}. \tag{22}$$

This result clearly demonstrates the crucial role of correlations for the variance of the conductance. These correlations are neglected in the traditional theory of Ericson fluctuations.

Of special interest is the $\kappa-$dependence of the correlations for a non-perfect coupling, in the regions $\kappa < 1$ and $\kappa > 1$. As one can see, the $\Lambda-$ and $\Sigma-$correlations have very different behavior, both for weak and strong intrinsic chaos. The remarkable fact is that, for the perfect coupling with $\kappa = 0$, the $\Lambda-$ and $\Sigma-$correlations have maximal values of opposite signs. These results may be important for the analysis of experimental nuclear or solid state data.

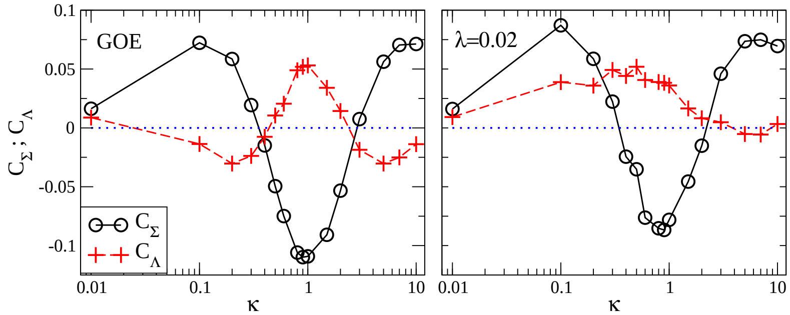

FIG. 6: Dependence of average correlations between cross sections on the coupling κ for the GOE case (left panel) and for a weak interaction with $\lambda = 0.02$ (right panel). Circles stand for the C_Σ and for the C_Λ correlations (refer to text for details). Both panels are on the same scale for $n = 6$ and $m = 12$.

F. Conclusions

In conclusion, we have studied the statistics of resonance widths and cross sections for a fermion system coupled to open decay channels. We used the effective non-Hermitian Hamiltonian that reflects exact quantum-mechanical features of many-body dynamics in an open interacting system. Main attention was paid to various signatures of the crossover from isolated to overlapping resonances in dependence on the strength of inter-particle interaction modeled by the two-body random matrices in the intrinsic part of the non-Hermitian Hamiltonian.

In the framework of our model we have tested main predictions of the Ericson fluctuation theory. We have shown that the assumption that the fluctuations of the resonance widths become negligible for a large number of channels, is invalid in the overlapping regime. We found that the fluctuations of resonance widths increase along with the coupling to the continuum, resulting in the divergence of the relative fluctuation of the widths (the ratio of the variance to the square of the average width) at the transition point $\kappa = 1$ for any number of channels.

We have also shown that, in agreement with previous studies, the correlation length differs from the average width for any number of channels. On the other hand, the Weisskopf relation (16) that connects the correlation length of the cross section to the transmission coefficient, works, for a large number of channels, at any value of the intrinsic interaction strength λ. In many situations the data show that increase of λ suppresses the fluctuations in the continuum. This can be understood qualitatively as a manifestation of many-body chaos that makes all internal states uniformly mixed [38].

Our results can be applied to any many-fermion system coupled to the continuum of open decay channels. The natural applications first of all should cover neutron resonances in nuclei, where rich statistical material was accumulated but the transitional region from isolated to overlapped resonances was not studied in detail. Other open mesoscopic systems, for example, quantum dots and quantum wires, should be analyzed as well in the crossover region.

It was our special interest to study the mesoscopic fluctuations of the conductance in the dependence on the coupling strength and degree of intrinsic chaos. In particular, we have analyzed how the correlations between different cross sections influence the variance of the conductance fluctuations. Our data manifest that these correlations determine the universal value 1/8 (see, for example, [37, 39]) for the conductance variance in the perfect coupling regime, $\kappa = 1$, and for very strong interaction between particles. Thus, we demonstrate that the Ericson theory of cross-section fluctuation that neglects these correlations, is insufficient for the description of the mesoscopic fluctuations.

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