Topics in Continuum Mechanics applied to Biology/1:
Introduction

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Contents of the course

- Introduction of Continuum Mechanics and Biomechanics
- Fluid models applied to blood flow
- Hyperalistic models for skeletal muscle tissue

References:

Y.C. Fung, *Biomechanics. Circulation*, Springer-Verlag, 1997.

G.P. Galdi et. al, *Hemodynamical Flows*, Birkhäuser, 2008.

M.E. Gurtin, *An introduction to continuum mechanics*, Mathematics in Science and Engineering 158, Academic Press, New York-London, 1981.

R.W. Ogden, *Non-linear Elastic Deformations*, Dover Publications, 1997.

G. Chagnon et al., *Hyperelastic Energy Densities for Soft Biological Tissues: A Review*, Journal of Elasticity (2015) 120.

Fluid dynamics - Elasticity

Continuum mechanics

From Wikipedia, the free encyclopedia

Continuum mechanics is a branch of mechanics that deals with the analysis of the kinematics and the mechanical behavior of materials modeled as a continuous mass rather than as discrete particles. The French mathematician Augustin-Louis Cauchy was the first to formulate such models in the 19th century. Research in the area continues today.

Continuum mechanics	
Laws	[show]
Solid mechanics	[show]
Fluid mechanics	[show]
Rheology	[show]
Scientists	[show]

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Fluid:

> *"a substance, as a liquid or gas, that is capable of flowing and that changes its shape at a steady rate when acted upon by a force tending to change its shape"*

Fluid dynamics - Elasticity

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> *"a substance, as a liquid or gas, that is capable of flowing and that changes its shape at a steady rate when acted upon by a force tending to change its shape"*

Solid:

> *"a body or object having three dimensions (length, breadth, and thickness)"*

Fluid dynamics - Elasticity

Εύρηκα

Fluid dynamics - Elasticity

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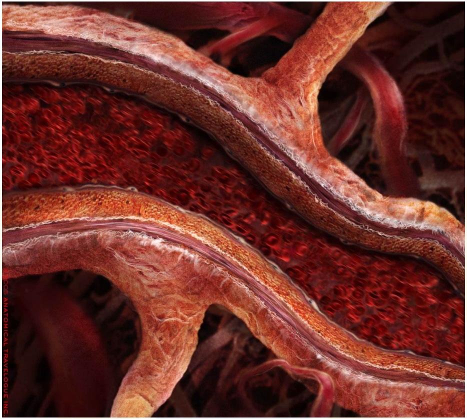
Fluid dynamics - Elasticity

Fluid dynamics - Elasticity

Fluid dynamics - Elasticity

Fluid dynamics - Elasticity



Biomechanics

Study of the motion of biological fluids and structures in any contents.

Biomechanics

Study of the motion of biological fluids and structures in any contents.

- Blood circulation
- Cardiovascular System
- Air flow in the respiratory system
- Flow in the eye
- Muscle and tendon tissue

What is Biomechanics useful for?

- **Pure physiology**: understanding how animals, and in particular humans, work.
- **Pathophysiology**: understanding why they might go wrong. In other words understanding the origins and development of diseases.
- **Diagnosis**: recognising diseases from possibly non-traumatic measurements.
- **Cure**: providing support to surgery and to the design of prosthetic devices.

Peculiarities of physiological fluid flows

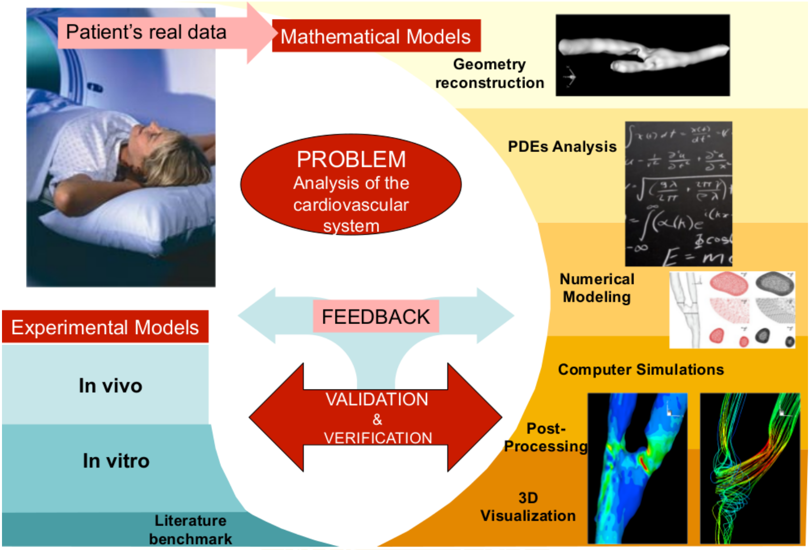
Thomas Young (1808):

> *"The mechanical motions, which take place in animal body, are regulated by the same general laws as the motion of inanimate bodies . . . and it is obvious that the enquiry, in what matter and in what degree, the circulation of the blood depends on the muscular and elastic powers of the heart and of the arteries, . . . , must become simply a question belonging to the most refined departments of the theory of hydraulics."*

Peculiarities of physiological fluid flows

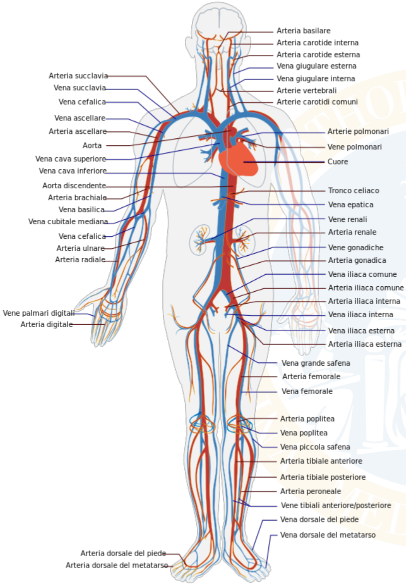
There are some key features which characterise physiological flows.

- **Pulsatility**: In most cases physiological flows are highly unsteady and are often pulsatile (e.g. flow in the systemic arteries or in the respiratory system ...).
- **Complex geometries**: Typically physiological flows take place in very complex geometries. In order to study the problems by analytical means it is therefore necessary to idealise the geometry in a suitable manner. It is a research challenge of recent years to perform numerical simulations on real geometries.
- **Deformability**: Not only the geometry of the flow domain might be complex but it also often varies in time. This typically induces great complication in the mathematical analysis. Often the problem to be solved is effectively a solid-fluid interaction.
- **Low Reynolds number flows**: In many cases of physiological interest (but by no means always) the Reynolds number of the flow is fairly low and this allows simplifying the equations.



Patient's real data
Mathematical Models
Geometry reconstruction
PDEs Analysis
Numerical Modeling
Computer Simulations
Post-Processing
3D Visualization
PROBLEM
Analysis of the cardiovascular system
FEEDBACK
VALIDATION
&
VERIFICATION
Experimental Models
In vivo
In vitro
Literature benchmark

Cardiovascular system



The circulatory system permits blood to circulate and transport nutrients (such as amino acids and electrolytes), oxygen, carbon dioxide, hormones, and blood cells to and from the cells in the body to provide nourishment and help in fighting diseases, stabilize temperature and pH, and maintain homeostasis.

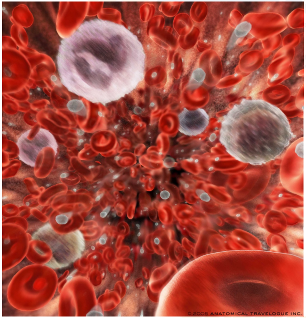
Important data

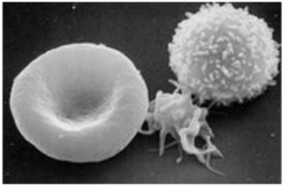
Vessel	number	diam [cm]	cross sectional area [cm^2]	wall thickness [cm]	mean pressure [KPa]	mean velocity [cm s^{-1}]
Aorta	1	3	7	2×10^{-1}	12.5	12
Arteries	8×10^{3}	10^{-1}	8×10^{-3}	10^{-1}	12	45
Arterioles	10^{7}	5×10^{-3}	2×10^{-5}	2×10^{-3}	7	5
Capillaries	10^{10}	8×10^{-4}	5×10^{-7}	10^{-4}	3	0.1
Venules	4×10^{7}	10^{-2}	7.9×10^{-5}	2×10^{-4}	1.5	2
Veins	8×10^{3}	1.8×10^{-1}	10^{-1}	5×10^{-2}	1	10
Vena cava	2	3	6	1.5×10^{-1}	0.5	14

Blood composition

Blood is a suspension of

- erythrocytes (RBCs, $\sim 2-8\mu m$ (volume $200\mu m^3$))
- leukocytes (WBCs)
- cells and platelets

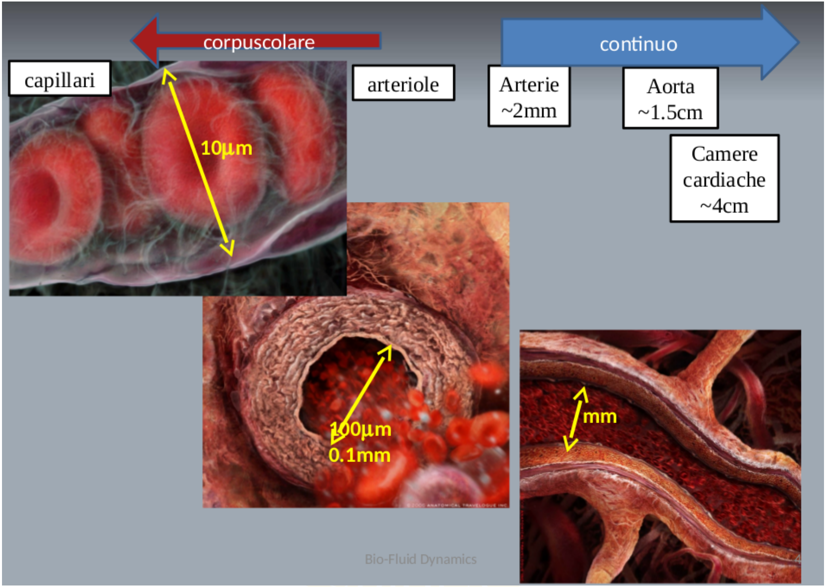




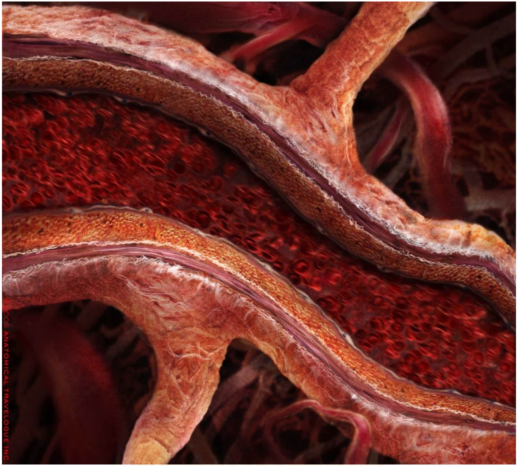
Blood composition

	Number/ mm^3	Shape (unstressed)	Size μm (unstressed)	Volume Conc.
erythrocytes	4-6 x 10^6	Biconcave discs with no nuclei	8-1-3	45%
leukocytes	4-11 x10^3	Roughly spherical	7-22	1%
platelets	2.5-5 x 10^5	Discoid cell fragments	2-4	

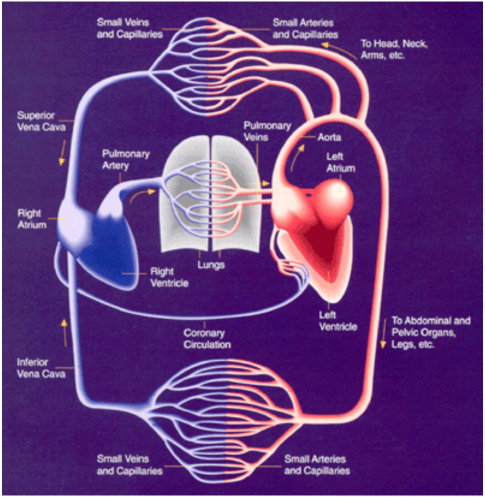
Plasma: 90-92% water + proteins, ions, organic salts.


corpuscolare
continuo
capillari
arteriole
Arterie
~2mm
Aorta
~1.5cm
Camere
cardiache
~4cm
10µm
100µm
0.1mm
mm
Bio-Fluid Dynamics

In the heart and large vessel, the scales of motion are so large (with respect to cell size) that blood is a homogeneous fluid.



Elasticity - Fluid dynamics

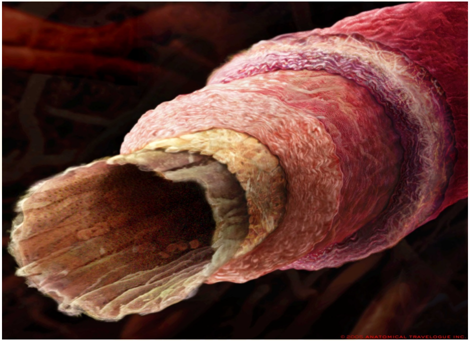


Elasticity: heart, veins, (skeletal muscle, . . .)

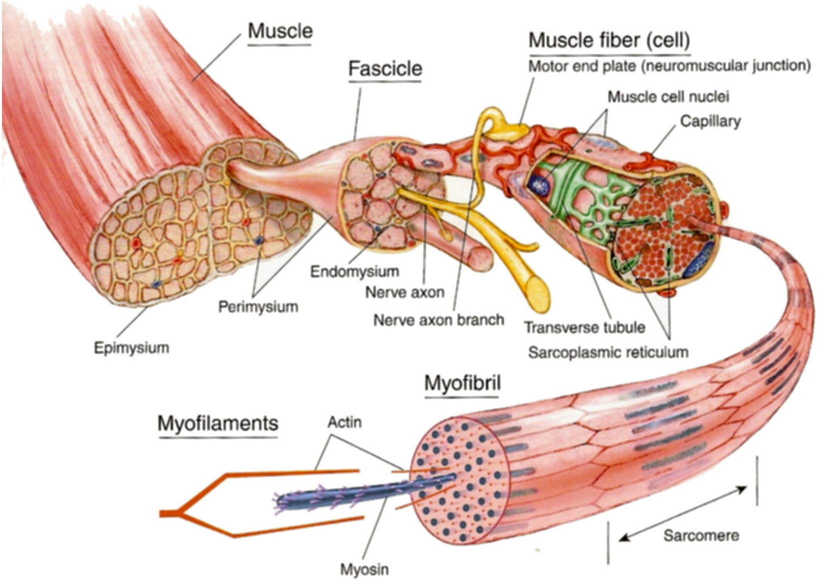
Fluid dynamics: blood, (air, aqueous humour, . . .)

Tissue

TISSUE: collection of elastic cells (or passive, or contractile)
looking at the collective behavior individual cells are not relevant and the tissue is more like a continuous material that deforms when a stress is applied (vessel that deforms because of blood pressure)



Skeletal muscle hierarchy



From the mathematical point of view (Eulerian)

$$\frac{d\rho}{dt} + \rho \,\mathrm{div}\, \mathbf{v} = 0 \tag{1}$$

$$\rho \frac{d\mathbf{v}}{dt} = \rho \mathbf{b} + \mathrm{div}\, \mathsf{T} \tag{2}$$

$$\mathsf{T} \quad \text{is symmetric} \tag{3}$$

From the mathematical point of view (Lagrangian)

$$\rho \det \mathsf{F} - \rho_0 = 0 \tag{4}$$

$$\rho_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} = \rho_0 \mathbf{B} + \operatorname{Div} \mathsf{P} \tag{5}$$

$$\mathsf{P}\mathsf{F}^T \quad \text{is symmetric} \tag{6}$$

Newtonian incompressible fluid

$$\mathsf{T} = -p\mathsf{I} + 2\mu\mathsf{D}\,; \tag{7}$$

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Newtonian incompressible fluid

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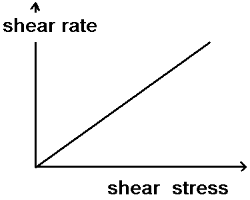
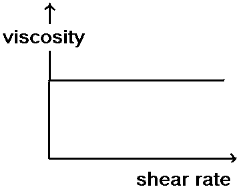
$$\mathsf{D} = \frac{1}{2}\left[\left(\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\right) + \left(\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\right)'\right].$$

A Newtonian liquid is a fluid in which the viscous stresses arising from its flow, at every point, are linearly proportional to the local strain rate: the constant of proportionality is called **viscosity**.

The viscosity of a fluid expresses its resistance to shearing flows.

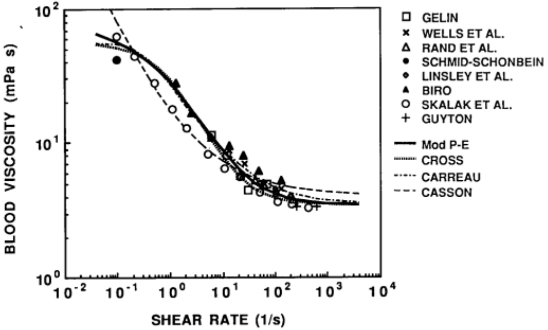
Newtonian incompressible fluid





Shear rate: $\dot{\gamma} := 2\mathsf{D}$
Shear stress (*viscosity tensor*): $\tau := \mathsf{T} + p\mathsf{I} =: \mathsf{V}$

But . . .

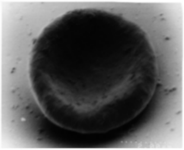


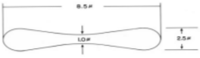
BLOOD VISCOSITY (mPa s)
10^2
10^1
10^0
10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 10^4
SHEAR RATE (1/s)
GELIN
WELLS ET AL.
RAND ET AL.
SCHMID-SCHONBEIN
LINSLEY ET AL.
BIRO
SKALAK ET AL.
GUYTON
Mod P-E
CROSS
CARREAU
CASSON

Non-Constant Viscosity

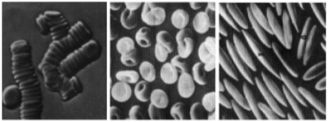
RBC aggregation and deformability

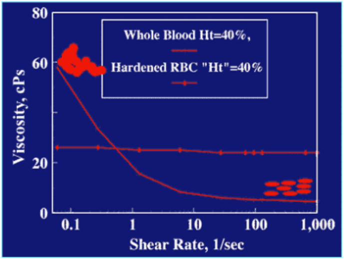
Shear -Thinning





Courtesy of Prof. K.B. Chandran, University of Iowa.





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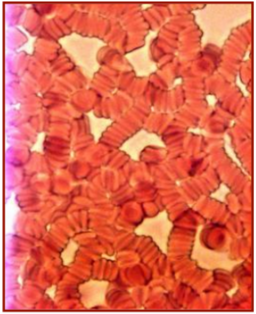
Hematocrit
Osmotic Pressure
Plasma Composition
…………………..

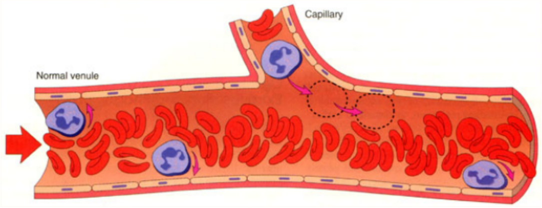
Non-Newtonian blood behavior/2

Rouleaux aggregation



Red blood cells aggregate as in stack of coins

Fåhraeus-Lindquist effect



In small vessels (below 1mm radii) red blood cells move toward the central part of the vessel, and blood viscosity shifts toward plasma viscosity (much lower)

Non-Newtonian blood behavior/3

Non-Linear Viscoelasticity

➔ Elastic behavior of RBC (elongation and distortion)

➔ Formation and distortion of the rouleaux

Non-Linear Creeping Stress

Normal Stress Effects

Viscoelastic parameters *experimentally measured* *e.g.* with unsteady flow in capillary tube viscometers

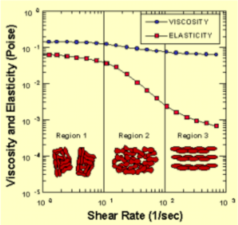


Fig. The shear rate dependence of normal human blood at 2Hz and 22°C [Vilastic Sc. Inc]

Oscillatory and pulsatile flow analysis

Haematocrit
Temperature
Time (Thixotropy)
Experimental factors
Plasma viscosity
…………………..

Viscoelastic effects are only substantial at low shear rates

Non-Newtonian effects

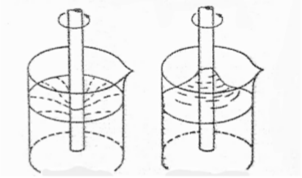
Thixotropy: due to the finite time required for the formation and breakdown of the rouleaux. It is a function of shear rate.

Yield-Stress: some experiments show that blood can resist shear, behaving rigidly, until a critical level of stress is reached (the yield stress). Above this value blood appears to flow like a fluid.

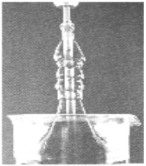
Non-Newtonian phenomena/1

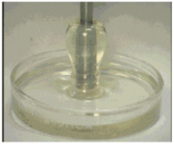
The "Weissenberg Effect" or "Rod-Climbing" Experiment

(Viscoelastic effect - due to normal stress differences)



Newtonian Fluid **Polymeric Fluid**





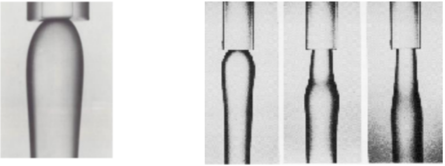
- The fluid is high-molecular weight polyisobutylene in low-molecular weight solvent of the same chemical nature.
- A rod is rotating is a dish of viscoelastic liquid and the liquid climbs up the rod.
- When the normal stress component in the circular direction of flow due to rotation of the rod is greater in magnitude than the two mutually perpendicular components, a tension in the flow direction results which increases as one approaches the surface of the rotating shaft
- A Newtonian liquid would move towards the rim of the dish under the influence of inertia forces.

From: H.A. Barnes,J.F.Hutto. K.A. Walters, *An Introduction to Rheology*, Elsevier Sc., Amsterdam (1989), and D.V. Boger and K. Walters, *Rheological Phenomena in Focus*, Elsevier Sc., (1993)]

Non-Newtonian phenomena/2

Post-Extrusion Effects: Extrudate Swell or "Die Swell"

(Viscoelastic effect mainly due to normal stress differences)

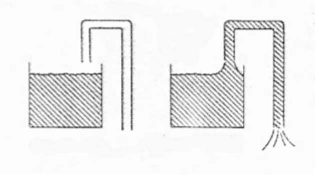


- **Delayed die swell** for a 5% aqueous solution of polyacrylamide.
  (a) Normal die swell. (b) Moderate flow strength. (c) Strong flow.

The liquid falls under gravity on exiting the capillary. For fully developed flow of a viscoelastic fluid in the tube, a tension along the streamlines associated with the normal stresses is present. As the flow strength increases, corresponding to **increasing Reynolds number**, it will relax the tension along the streamlines by contracting in a longitudinal direction; there is a.conflict between fluid inertia and elasticity. This results in lateral expansion of the liquid, giving rise to the die-swell phenomenon; the swelling phenomenon is delayed as a result of fluid inertia.

Non-Newtonian phenomena/3

The " Tubeless Siphon" or "Open-Siphon Effect" and Fano Flow
(Extensional Viscoelastic effect)

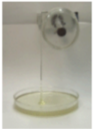


Newtonian Fluid **Polymeric Fluid**

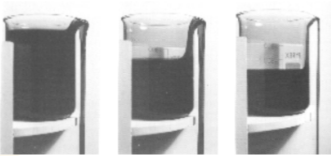
Test sample: 0.75% aqueous solution of Polyox WSR 301
• Only the slightest spilling will part empty the beaker.

When a liquid containing large molecules of high aspect ratio particles is stretched, the molecules and/or the particles align in the direction of stretching process. This results in a substantial increase in the extensional viscosity of the liquids, especially at high extensional strain rates.

From: D.V. Boger and K. Walters,
Rheological Phenomena in Focus, Elsevier Sc., (1993)]

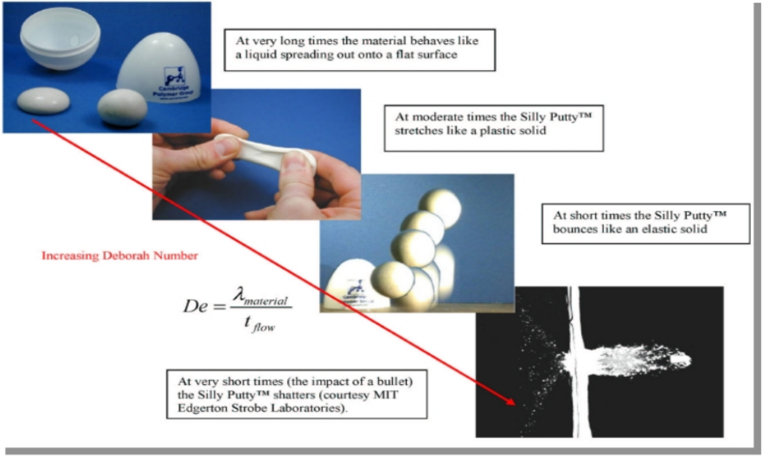


Thread produced by raising the dish, with the surface of the liquid just below the orifice. Fluid is drawn into the chamber by applying a vacuum pressure and the reservoir is then slowly lowered to produce the phenomenon.
• The vacuum chamber is now replaced by a rotating drum

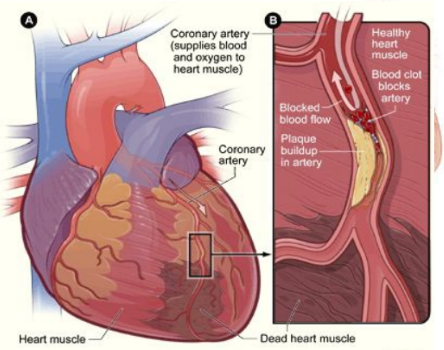


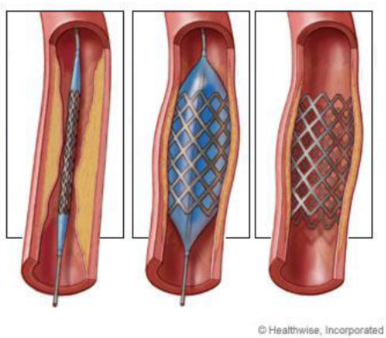
Very special behavior

NON-NEWTONIAN FLOW PHENOMENA



Turbolence/1





Turbolence/2