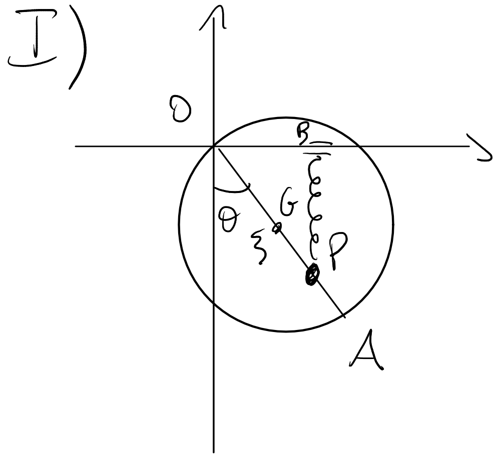
MA 6/6/25

$\xi \in [0, 2R]$

I)



$(G-O) = R\sin\theta,\ -R\cos\theta$

$(P-O) = \xi\sin\theta,\ -\xi\cos\theta$

$$U = -mg\,y_G - mg\,y_P - \frac{1}{2}k\,|P-B|^2$$

$$= mgR\cos\theta + mg\xi\cos\theta - \frac{1}{2}k\xi^2\cos^2\theta$$

$$\begin{cases} \dfrac{\partial U}{\partial \xi} = mg\cos\theta - k\xi\cos^2\theta \\ \dfrac{\partial U}{\partial \theta} = -mgR\sin\theta - mg\xi\sin\theta + k\xi^2\sin\theta\cos\theta \end{cases}$$

$\cos\theta = 0 \qquad \theta = \frac{\pi}{2},\ \frac{3}{2}\pi$

$mg = k\xi\cos\theta$

$\theta = \frac{\pi}{2} \qquad -mgR - mg\xi = 0 \qquad$ N.A

$\theta = \frac{3}{2}\pi \qquad -mgR - mg\xi = 0 \qquad$ N.A.

$mg = k\xi\cos\theta$

$\sin\theta = 0 \qquad \theta = 0, \pi$

$-mgR - mg\xi + \underbrace{k\xi^2\cos\theta}_{\xi mg} = 0 \Rightarrow mgR = 0$ N.A.

$\vartheta = 0 \qquad k\xi = mg \qquad \xi = \frac{mg}{k} < 2R \qquad \lambda < 2$

$$\boxed{\left(\frac{mg}{k}, 0\right) \quad \text{per } \lambda < 2}$$

$\vartheta = \pi \qquad -k\xi = mg \quad$ N.A.

$$H = \begin{bmatrix} -k\cos^2\vartheta & -mg\sin\vartheta + 2k\xi\sin\vartheta\cos\vartheta \\ '' & -mgR\cos\vartheta - mg\xi\cos\vartheta + k\xi^2(\cos^2\vartheta - \sin^2\vartheta) \end{bmatrix}$$

$$H\left(\frac{mg}{k}, 0\right) = \begin{bmatrix} -k & 0 \\ 0 & -mgR - \cancel{\frac{m^2g^2}{k}} + \cancel{k\frac{m^2g^2}{k^2}} \end{bmatrix}$$

$\Rightarrow \left(\frac{mg}{k}, 0\right) \quad$ STABILE

2) $\xi = 0 \qquad \begin{cases} \frac{\partial U}{\partial \xi} \leq 0 \\ \frac{\partial U}{\partial \vartheta} = 0 \end{cases}$

$$\frac{\partial U}{\partial \xi} = mg\cos\vartheta - k\cancel{\xi\cos^2\vartheta} \leq 0$$

$$\frac{\partial U}{\partial \vartheta} = -mgR\sin\vartheta - \cancel{mg\xi\sin\vartheta} + \cancel{k\xi^2\sin\vartheta\cos\vartheta} = 0$$

$\sin\theta = 0 \qquad \theta = 0, \pi$

$mg\cos\theta \le 0 \qquad \theta = \pi$

$(0, \pi)$ eq. confine

$\xi = 2R \qquad \frac{\partial U}{\partial \xi} \ge 0$

$\frac{\partial U}{\partial \theta} = 0$

$$\frac{\partial U}{\partial \xi} = mg\cos\theta - k\overset{2R}{\xi}\cos^2\theta \ge 0$$

$$\frac{\partial U}{\partial \theta} = -mgR\sin\theta - mg\overset{2R}{\xi}\sin\theta + k\overset{4R^2}{\xi^2}\sin\theta\cos\theta = 0$$

$$-mgR\sin\theta - 2mgR\sin\theta + 4kR^2\sin\theta\cos\theta = 0$$

$\sin\theta = 0 \qquad \theta = 0, \pi$

$-3mgR + 4kR^2\cos\theta = 0$

$\theta = 0 \qquad mg - 2kR \ge 0 \qquad \lambda \ge 2$

$(2R, 0)$ eq. confine per $\lambda \ge 2$

$\theta = \pi \qquad -mg - 2kR \ge 0 \qquad$ MAI

$4kR^{\cancel{2}}\cos\theta = 3mg\cancel{R} \Rightarrow 2kR\cos\theta = \frac{3}{2}mg$

$\cos\theta(mg - 2kR\cos\theta) \ge 0$

$\cos\theta\left(mg - \frac{3}{2}mg\right) \ge 0 \qquad \cos\theta \le 0$

$\cos\theta = \frac{3}{4}\lambda \qquad$ MAI

3) $K_{disco} = \frac{1}{2} \cdot \frac{3}{2} m R^2 \dot{\theta}^2 = \frac{3}{4} m R^2 \dot{\theta}^2$

$$\vec{v}_p = \left(\dot{\xi} \sin\theta + \xi \dot{\theta} \cos\theta ,\ -\dot{\xi} \cos\theta + \xi \dot{\theta} \sin\theta\right)$$

$$|v_p|^2 = \dot{\xi}^2 + \xi^2 \dot{\theta}^2$$

$$K_{punto} = \frac{1}{2} m \left(\dot{\xi}^2 + \xi^2 \dot{\theta}^2\right)$$

$$K = \frac{3}{4} m R^2 \dot{\theta}^2 + \frac{1}{2} m \dot{\xi}^2 + \frac{1}{2} m \xi^2 \dot{\theta}^2$$

4) $$\mathbb{K} = \begin{bmatrix} m & 0 \\ 0 & m\left(\xi^2 + \frac{3}{2} R^2\right) \end{bmatrix}$$

$$H = \begin{bmatrix} -k & 0 \\ 0 & -mgR \end{bmatrix}$$

$$\tilde{\mathcal{L}} = \frac{1}{2} \begin{bmatrix} \dot{\xi} & \dot{\theta} \end{bmatrix} \mathbb{K} \begin{bmatrix} \dot{\xi} \\ \dot{\theta} \end{bmatrix} - \frac{1}{2} \begin{bmatrix} \xi - \frac{mg}{k} & \theta \end{bmatrix} H \begin{bmatrix} \xi - \frac{mg}{k} \\ \theta \end{bmatrix}$$

$$= \frac{1}{2} \left(m \dot{\xi}^2 + m\left(\xi^2 + \frac{3}{2} R^2\right) \dot{\theta}^2 + k\left(\xi - \frac{mg}{k}\right)^2 + mgR\theta^2 \right)$$

$\square$

II) $$\begin{cases} Q = \sqrt{2p} \sin q \\ P = \sqrt{p} \cos q \end{cases}$$

$$[Q, P] = \sqrt{\alpha \cancel{p}}\cos q \, \frac{1}{2\cancel{\sqrt{p}}}\cos q + \frac{\sqrt{\alpha}}{2\cancel{\sqrt{p}}}\sin q \, \cancel{\sqrt{p}}\sin q$$

$$= \frac{\sqrt{\alpha}}{2}\left(\cos^2 q + \sin^2 q\right) = \frac{\sqrt{\alpha}}{2} = 1$$

$$\Rightarrow \sqrt{\alpha} = 2 \qquad \boxed{\alpha = 4}$$

$$\begin{cases} p = \dfrac{\partial F_2}{\partial q} \\ Q = \dfrac{\partial F_2}{\partial P} \end{cases} \qquad Q = 2\,\frac{P}{\cos q}\sin q = 2P \operatorname{tg} q \qquad p = \frac{P^2}{\cos^2 q}$$

$$F_2 = P^2 \operatorname{tg} q + g(P)$$

$$p = \frac{\partial F_2}{\partial q} = P^2 \frac{1}{\cos^2 q} + g'(P) = \frac{P^2}{\cos^2 q}$$

$$\Rightarrow g'(P) = 0 \qquad g \equiv \text{cost}$$

$$F_2(q, P) = P^2 \operatorname{tg} q + \text{cost}.$$

$\square$