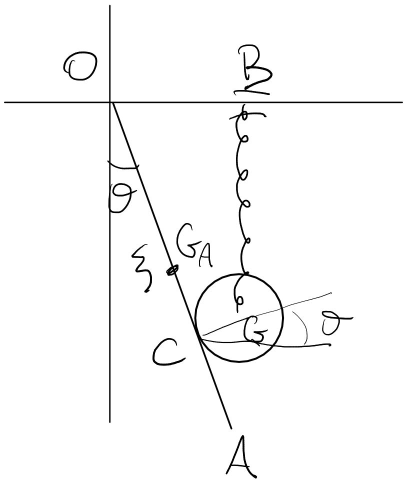

$$\xi \in [0,\ 8R] \qquad \lambda = \frac{mg}{kR}$$

$$(G_A - O) = 4R\sin\vartheta,\ -4R\cos\vartheta$$

$$(C - O) = \xi\sin\vartheta,\ -\xi\cos\vartheta$$

$$(G - C) = R\cos\vartheta,\ R\sin\vartheta$$

$$(G - O) = \xi\sin\vartheta + R\cos\vartheta,\ -\xi\cos\vartheta + R\sin\vartheta$$

$$U = -mg\, y_{G_A} - mg\, y_G - \frac{1}{2}k\,|G-B|^2$$

$$= 4mgR\cos\vartheta + mg\xi\cos\vartheta - mgR\sin\vartheta - \frac{1}{2}k(-\xi\cos\vartheta + R\sin\vartheta)^2$$

$$\begin{cases} \dfrac{\partial U}{\partial \xi} = mg\cos\vartheta + k(-\xi\cos\vartheta + R\sin\vartheta)\cos\vartheta \\ \dfrac{\partial U}{\partial \vartheta} = -4mgR\sin\vartheta - mg\xi\sin\vartheta - mgR\cos\vartheta - k(-\xi\cos\vartheta + R\sin\vartheta)(\xi\sin\vartheta + R\cos\vartheta) \end{cases}$$

$$\cos\vartheta\left[mg + k(-\xi\cos\vartheta + R\sin\vartheta)\right] = 0$$

- $\cos\vartheta = 0 \qquad \vartheta = \frac{\pi}{2},\ \frac{3}{2}\pi$

$$\vartheta = \frac{\pi}{2} \qquad -4mgR - mg\xi - kR\xi = 0$$

$$\xi = -\frac{4mgR}{mg + kR} \qquad \text{N.A.}$$

$\vartheta = \frac{3}{2}\pi \qquad 4mgR + mg\xi - kR\xi = 0$

$$\xi = \frac{4mgR}{kR - mg} = \boxed{\frac{4\lambda R}{1-\lambda}}$$

$$kR > mg \Rightarrow \lambda < 1$$

$$0 < \frac{4\lambda R}{1-\lambda} < 8R \Rightarrow \lambda < 1$$

$$4\lambda < 8 - 8\lambda$$

$$\boxed{\lambda < \frac{8}{12} = \frac{2}{3}}$$

$$-k(-\xi\cos\vartheta + R\sin\vartheta) = mg$$

$$-4mgR\sin\vartheta - \cancel{mg\xi\sin\vartheta} - \cancel{mgR\cos\vartheta}$$

$$+ mg(\cancel{\xi\sin\vartheta} + \cancel{R\cos\vartheta}) = 0$$

$$\sin\vartheta = 0 \qquad \vartheta = 0, \pi$$

$$\vartheta = 0 \qquad +k\xi = mg \qquad \boxed{\xi = \frac{mg}{k} = \lambda R \quad \lambda < 8}$$

$$\vartheta = \pi \qquad -k\xi = mg \quad \text{N.A.}$$

$$\xi = \frac{4\lambda R}{1-\lambda}, \quad \vartheta = \frac{3}{2}\pi \qquad \lambda < \frac{2}{3}$$

$$\xi = \lambda R, \quad \vartheta = 0 \qquad \lambda < 8$$

$$H = \begin{bmatrix} -k\cos^2\vartheta & -mg\sin\vartheta + k(\xi\sin\vartheta + R\cos\vartheta)\cos\vartheta - k(-\xi\cos\vartheta + R\sin\vartheta)\sin\vartheta \\ '' & -4mgR\cos\vartheta - mg\xi\cos\vartheta + mgR\sin\vartheta - k(\xi\sin\vartheta + R\cos\vartheta)^2 + k(-\xi\cos\vartheta + R\sin\vartheta)^2 \end{bmatrix}$$

$$H\left(\frac{4\lambda R}{1-\lambda}, \frac{3}{2}\pi\right) = \begin{bmatrix} 0 & mg - kR \\ mg - kR & mgR - k\xi^2 + kR^2 \end{bmatrix}$$

$\det H < 0$ INST.

$$H(\lambda R, 0) = \begin{bmatrix} -k & kR \\ kR & -4mgR - mg\lambda R - kR^2 + k\lambda^2R^2 \end{bmatrix}$$

$$= \begin{bmatrix} -k & kR \\ kR & kR^2(-4\lambda - \cancel{\lambda^2} - 1 + \cancel{\lambda^2}) \end{bmatrix}$$

$$\det = k^2R^2(1+4\lambda) - k^2R^2 = k^2R^2(\cancel{1}+4\lambda-\cancel{1}) > 0$$

$tr < 0$ STABILE

$\xi = 0$

$$\begin{cases} mg\cos\vartheta + k(-\xi\cos\vartheta + R\sin\vartheta)\cos\vartheta \leq 0 \\ -4mgR\sin\vartheta - mg\xi\sin\vartheta - mgR\cos\vartheta \\ \qquad -k(-\xi\cos\vartheta + R\sin\vartheta)(\xi\sin\vartheta + R\cos\vartheta) = 0 \end{cases}$$

$$\begin{cases} mg\cos\vartheta + kR\sin\vartheta\cos\vartheta \leq 0 \\ -4mgR\sin\vartheta - mgR\cos\vartheta - kR^2\sin\vartheta\cos\vartheta = 0 \end{cases}$$

CONTI DIFFICILI

$\xi = 8R$

$$\begin{cases} mg\cos\vartheta + k(-8R\cos\vartheta + R\sin\vartheta)\cos\vartheta \geq 0 \\ -4mgR\sin\vartheta - 8mgR\sin\vartheta - mgR\cos\vartheta \\ \qquad -k(-8R\cos\vartheta + R\sin\vartheta)(8R\sin\vartheta + R\cos\vartheta) = 0 \end{cases}$$

CONTI DIFFICILI

$$K = \frac{1}{2} J_O^{asta} \dot{\vartheta}^2 + \frac{1}{2} \omega \cdot J_G^{disco} \omega + \frac{1}{2} m |v_G|^2$$

$$\omega = \dot{\vartheta} + \dot{\varphi} \qquad R\dot{\varphi} = -\dot{\xi} \qquad \omega^2 = \left(\dot{\vartheta} - \frac{\dot{\xi}}{R}\right)^2$$

$$(G-O) = \xi \sin\vartheta + R\cos\vartheta,\ -\xi\cos\vartheta + R\sin\vartheta$$

$$\Rightarrow\ v_G = \dot{\xi}\sin\vartheta + \xi\dot{\vartheta}\cos\vartheta - R\dot{\vartheta}\sin\vartheta,\ -\dot{\xi}\cos\vartheta + \xi\dot{\vartheta}\sin\vartheta + R\dot{\vartheta}\cos\vartheta$$

$$|v_G|^2 = \dot{\xi}^2 + \xi^2\dot{\vartheta}^2 + R^2\dot{\vartheta}^2 - 2R\dot{\xi}\dot{\vartheta}$$

$$J_O^{asta} = \frac{1}{3} m (8R)^2 = \frac{64}{3} m R^2$$

$$J_G^{disco} = \frac{1}{2} m R^2$$

$$K = \frac{1}{2} \cdot \frac{64}{3} m R^2 \dot{\vartheta}^2 + \frac{1}{2} \cdot \frac{1}{2} m R^2 \left(\dot{\vartheta} - \frac{\dot{\xi}}{R}\right)^2 + \frac{1}{2} m \left(\dot{\xi}^2 + \xi^2\dot{\vartheta}^2 + R^2\dot{\vartheta}^2 - 2R\dot{\xi}\dot{\vartheta}\right) =$$

$$\frac{m}{2}\left[\frac{64}{3} R^2\dot{\vartheta}^2 + \frac{1}{2} R^2\dot{\vartheta}^2 + \frac{1}{2}\dot{\xi}^2 - 2R\dot{\xi}\dot{\vartheta} + \dot{\xi}^2 + \xi^2\dot{\vartheta}^2 + R^2\dot{\vartheta}^2 - 2R\dot{\xi}\dot{\vartheta}\right]$$

$$= \frac{1}{2} m \left[\left(\frac{137}{6} R^2 + \xi^2\right)\dot{\vartheta}^2 + \frac{3}{2}\dot{\xi}^2 - 4R\dot{\xi}\right]$$

II

$$J_G = \begin{bmatrix} \frac{mR^2}{4} & & \\ & \frac{mR^2}{4} & \\ & & \frac{mR^2}{2} \end{bmatrix}$$

$$d = (-R, R, 0)$$
$$d^2 = 2R^2$$

$$d \otimes d = \begin{bmatrix} R^2 & -R^2 & 0 \\ -R^2 & R^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_O^1 = J_G + m \begin{bmatrix} R^2 & R^2 & 0 \\ R^2 & R^2 & 0 \\ 0 & 0 & 2R^2 \end{bmatrix} = \begin{bmatrix} \frac{5}{4}mR^2 & mR^2 & 0 \\ mR^2 & \frac{5}{4}mR^2 & 0 \\ 0 & 0 & \frac{5}{2}mR^2 \end{bmatrix}$$

$$J_O^2 = J_O^1$$

$$J_O^{asta} = \begin{bmatrix} \frac{1}{12}m4R^2 & \frac{1}{12}m4R^2 & 0 \\ \frac{1}{12}m4R^2 & \frac{1}{12}m4R^2 & 0 \\ 0 & 0 & \frac{1}{6}m4R^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3}mR^2 & \frac{1}{3}mR^2 & 0 \\ \frac{1}{3}mR^2 & \frac{1}{3}mR^2 & 0 \\ 0 & 0 & \frac{2}{3}mR^2 \end{bmatrix}$$

$$\frac{5}{2} + \frac{1}{3} = \frac{15+2}{6}$$

$$J^{TOT} = 2J_O^1 + J_O^{asta}$$

$$J^{TOT} = \begin{bmatrix} \frac{17}{6}mR^2 & \frac{7}{3}mR^2 & 0 \\ \frac{7}{3}mR^2 & \frac{17}{6}mR^2 & 0 \\ 0 & 0 & \frac{17}{3}mR^2 \end{bmatrix}$$

$\square$