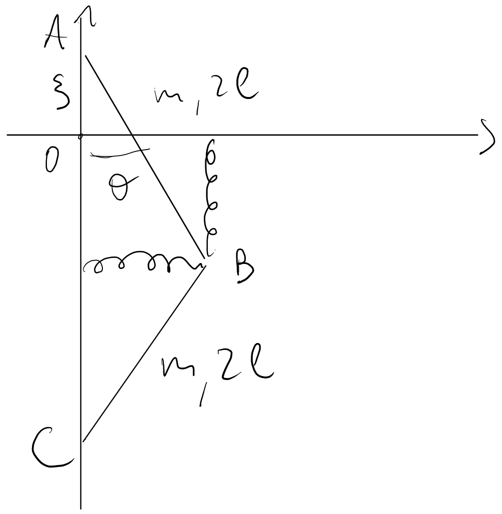
18/7/25



$$(G_1 - O) = (G_1 - A) + (A - O)$$
$$= l\sin\theta,\ \xi - l\cos\theta$$
$$(B - O) = (B - A) + (A - O)$$
$$= 2l\sin\theta,\ \xi - 2l\cos\theta$$
$$(G_2 - O) = (G_2 - B) + (B - O)$$
$$= l\sin\theta,\ \xi - 3l\cos\theta$$

$$U = -mg(y_{G_1} + y_{G_2}) - \frac{1}{2}k\,|B-O|^2$$
$$= -mg(2\xi - 4l\cos\theta) - \frac{1}{2}k(\xi^2 - 4l\xi\cos\theta) + \text{cost}$$
$$= -2mg\xi + 4mgl\cos\theta - \frac{1}{2}k\xi^2 + 2kl\xi\cos\theta + c$$

$$\frac{\partial U}{\partial \xi} = -2mg - k\xi + 2kl\cos\theta \quad = 0$$
$$\frac{\partial U}{\partial \theta} = -4mgl\sin\theta - 2kl\xi\sin\theta \quad = 0$$
$$2l\sin\theta(2mg + k\xi) = 0$$

$\theta = 0$ $\qquad -2mg - k\xi + 2kl = 0$
$$\xi = 2l - \frac{2mg}{k} = 2l(1-\lambda)$$

$\theta = \pi$ $\qquad -2mg - k\xi - 2kl = 0$
$$\xi = -2l - \frac{2mg}{k} = -2l(1+\lambda)$$

$P_1\,(2l(1-\lambda), 0)$ $\qquad P_2\,(-2l(1+\lambda), \pi)$

$$2mg + k\xi = 0 \qquad \xi = -\frac{2mg}{k} = -2\lambda\ell$$

$$\cancel{-2mg} + \cancel{2mg} + 2k\ell\cos\theta = 0$$

$$\cos\theta = 0 \qquad \theta = \frac{\pi}{2},\ \frac{3}{2}\pi$$

$$P_3 = \left(-2\lambda\ell,\ \frac{\pi}{2}\right) \qquad P_4\left(-2\lambda\ell,\ \frac{3}{2}\pi\right)$$

$$H = \begin{bmatrix} -k & -2k\ell\sin\theta \\ -2k\ell\sin\theta & -4mg\ell\cos\theta - 2k\ell\xi\cos\theta \end{bmatrix}$$

$$H(P_1) = \begin{bmatrix} -k & 0 \\ 0 & -4mg\ell - 4k\ell^2(1-\lambda) \end{bmatrix}$$

$$-4k\ell^2(\cancel{\lambda} + 1 - \cancel{\lambda})$$

$$-4k\ell^2 < 0$$

STAB.

$$H(P_2) = \begin{bmatrix} -k & 0 \\ 0 & +4mg\ell - 4k\ell^2(1+\lambda) \end{bmatrix}$$

STAB.

$$4k\ell^2(\cancel{\lambda} - 1 - \cancel{\lambda}) < 0$$

$$H(P_{3,4}) = \begin{bmatrix} -k & \mp 2k\ell \\ \mp 2k\ell & 0 \end{bmatrix}$$

INST.

$$v_{G_1} = \ell\dot\vartheta\cos\vartheta,\quad \dot\xi + \ell\dot\vartheta\sin\vartheta$$

$$v_{G_2} = \ell\dot\vartheta\cos\vartheta,\quad \dot\xi + 3\ell\dot\vartheta\sin\vartheta$$

$$|v_{G_1}|^2 = \ell^2\dot\vartheta^2 + \dot\xi^2 + 2\ell\dot\xi\dot\vartheta\sin\vartheta$$

$$|v_{G_2}|^2 = \dot\xi^2 + \ell^2\dot\vartheta^2\underbrace{(\cos^2\vartheta + 9\sin^2\vartheta)}_{1+8\sin^2\vartheta} + 6\ell\dot\xi\dot\vartheta\sin\vartheta$$

$$K = \frac{1}{2}m\left(2\dot\xi^2 + \ell^2\dot\vartheta^2(2+8\sin^2\vartheta) + 8\ell\dot\xi\dot\vartheta\sin\vartheta\right)$$

$$+ \frac{1}{2}\quad\frac{2}{3}m\ell^2\quad\dot\vartheta^2$$

$$= \frac{1}{2}m\left(2\dot\xi^2 + \left(\frac{8}{3}+8\sin^2\vartheta\right)\ell^2\dot\vartheta^2 + 8\ell\dot\xi\dot\vartheta\sin\vartheta\right)$$

$$\mathbb{K} = \begin{bmatrix} 2m & 4m\ell\sin\vartheta \\ 4m\ell\sin\vartheta & \left(\frac{8}{3}+8\sin^2\vartheta\right)m\ell^2 \end{bmatrix}$$

$$\mathbb{K}(P_1) = \begin{bmatrix} 2m & 0 \\ 0 & \frac{8}{3}m\ell^2 \end{bmatrix} \qquad H(P_1) = \begin{bmatrix} -k & 0 \\ 0 & -4k\ell^2 \end{bmatrix}$$

$$\tilde{\mathcal{L}} = \frac{1}{2}\left(2m\dot\xi^2 + \frac{8}{3}m\ell^2\dot\vartheta^2\right) + \frac{1}{2}\left[k(\xi - 2\ell(1-\lambda))^2 + 4k\ell^2\vartheta^2\right]$$

$\square$

II) lamina quadrata

$$\begin{bmatrix} \frac{ml^2}{3} & & \\ & \frac{ml^2}{3} & \\ & & \frac{2}{3}ml^2 \end{bmatrix}$$

$$d = (l\sqrt{2}, l\sqrt{2}, 0) \qquad d^2 = 4l^2$$

$$d^2 I - d \otimes d = \begin{bmatrix} 2l^2 & -2l^2 & 0 \\ -2l^2 & 2l^2 & 0 \\ 0 & 0 & 4l^2 \end{bmatrix}$$

$$J_Q = \begin{bmatrix} \frac{7}{3}ml^2 & -2ml^2 & 0 \\ -2ml^2 & \frac{7}{3}ml^2 & 0 \\ 0 & 0 & \frac{14}{3}ml^2 \end{bmatrix}$$

$$J_{asta} = \begin{bmatrix} \frac{ml^2}{6} & -\frac{ml^2}{6} & 0 \\ -\frac{ml^2}{6} & \frac{ml^2}{6} & 0 \\ 0 & 0 & \frac{ml^2}{3} \end{bmatrix}$$

$$J_{tot} = \begin{bmatrix} \frac{29}{6}ml^2 & -\frac{25}{6}ml^2 & 0 \\ -\frac{25}{6}ml^2 & \frac{29}{6}ml^2 & 0 \\ 0 & 0 & \frac{29}{3}ml^2 \end{bmatrix}$$

$\square$