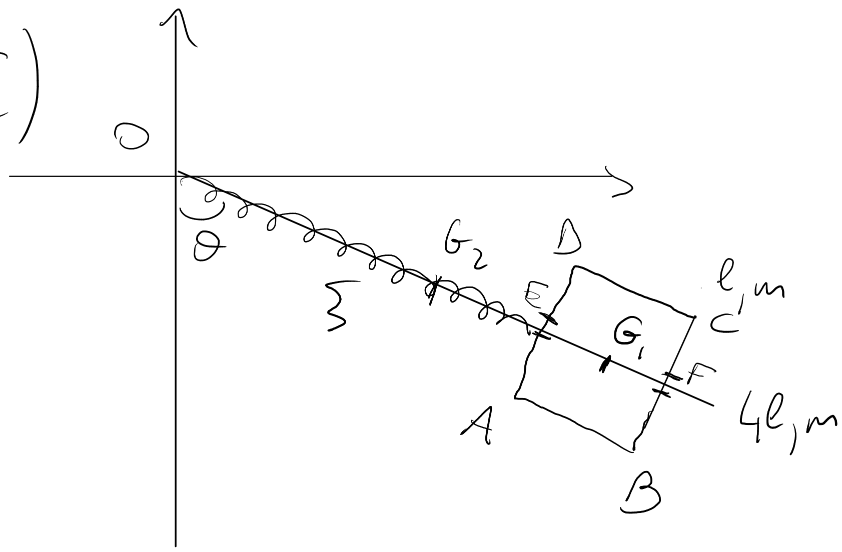
MA 8/6/26

I)



E, F ∈ asta

$\Rightarrow \quad 0 \le \xi \le 3l$

$(G_2 - O) = 2l\sin\theta, \ -2l\cos\theta$

$(G_1 - O) = (\xi + \frac{l}{2})\sin\theta, \ -(\xi + \frac{l}{2})\cos\theta$

$(E - O) = \xi\sin\theta, -\xi\cos\theta$

$U = 2mgl\cos\theta + mg(\xi + \frac{l}{2})\cos\theta - \frac{1}{2}k\xi^2$

$\frac{\partial U}{\partial \xi} = mg\cos\theta - k\xi = 0$

$\lambda = \frac{mg}{kl}$

$\frac{\partial U}{\partial \theta} = \left(-2mgl - mg\xi - mg\frac{l}{2}\right)\sin\theta = 0$

$\sin\theta = 0$ — $\theta = 0$; $\theta = \pi$

$\xi = \frac{mg}{k} = \lambda l \qquad \lambda < 3$

$\xi = -\frac{mg}{k}$ N.A.

$\xi = -2l - \frac{l}{2} < 0$ N.A.

$P(\lambda l, 0)$ per $\lambda < 3$

Stabilità:

$$\mathcal{H}f = \begin{bmatrix} -k & -mg\sin\vartheta \\ -mg\sin\vartheta & mg\left(-2l-\xi-\frac{l}{2}\right)\cos\vartheta \end{bmatrix}$$

$$\mathcal{H}f(P) = \begin{bmatrix} -k & 0 \\ 0 & \underbrace{mg\left(-2l-\lambda l-\frac{l}{2}\right)}_{<0} \end{bmatrix}$$ STABILE

Confine

$\xi = 0 \qquad \dot\xi \geq 0$

$$\begin{cases} mg\cos\vartheta \leq 0 \\ mg\left(-2l-\frac{l}{2}\right)\sin\vartheta = 0 \end{cases}$$

$\vartheta = 0 \qquad mg \leq 0 \qquad$ MAI

$\vartheta = \pi \qquad -mg \leq 0 \qquad$ sempre

$(0, \pi)$ di confine $\forall \lambda > 0$

$\xi = 3l \qquad \dot\xi \leq 0$

$$\begin{cases} mg\cos\vartheta \geq 3kl \\ mg\left(-2l-3l-\frac{l}{2}\right)\sin\vartheta = 0 \end{cases}$$

$\vartheta = 0 \qquad mg \geq 3kl \qquad \lambda \geq 3$

$\vartheta = \pi \qquad -mg \geq 3kl \qquad$ mai

$(3\ell, 0)$ di confine per $\lambda \geq 3$

Energia cinetica:

$$K_{asta} = \frac{1}{2}\,\frac{1}{3} m (4\ell)^2 \dot{\theta}^2 = \frac{1}{2} m \,\frac{16}{3} \ell^2 \dot{\theta}^2$$

K_{lamina} $\quad v_{G_1} = \dot{\xi} \sin\theta + \left(\xi + \frac{\ell}{2}\right)\dot{\theta} \cos\theta ,$

$$\dot{\xi} \cos\theta - \left(\xi + \frac{\ell}{2}\right)\dot{\theta} \sin\theta$$

$$|v_{G_1}|^2 = \dot{\xi}^2 + \left(\xi + \frac{\ell}{2}\right)^2 \dot{\theta}^2$$

$$K_{lamina} = \frac{1}{2} m \dot{\xi}^2 + \frac{1}{2} m \left(\xi + \frac{\ell}{2}\right)^2 \dot{\theta}^2 + \frac{1}{2}\,\frac{1}{6} m \ell^2 \dot{\theta}^2$$

$$= \frac{1}{2} m \left(\dot{\xi}^2 + \left[\left(\xi + \frac{\ell}{2}\right)^2 + \frac{\ell^2}{6}\right] \dot{\theta}^2 \right)$$

$$K_{tot} = \frac{1}{2} m \dot{\xi}^2 + \frac{1}{2} m \left[\left(\xi + \frac{\ell}{2}\right)^2 + \frac{\ell^2}{6} + \frac{16}{3}\ell^2 \right] \dot{\theta}^2$$

$$\mathbb{K}(P) = \begin{bmatrix} m & 0 \\ 0 & m\left[\left(\lambda\ell + \frac{\ell}{2}\right)^2 + \frac{\ell^2}{6} + \frac{16}{3}\ell^2\right] \end{bmatrix}$$

$\lambda = 1$ $\quad ml^2\left[\frac{9}{4} + \frac{1}{6} + \frac{16}{3}\right] = ml^2 \, \frac{27+2+64}{12}$

$$= \frac{93}{12} ml^2 = \frac{31}{4} ml^2$$

$$\mathbb{K} = \begin{bmatrix} m & 0 \\ 0 & \frac{31}{4} ml^2 \end{bmatrix} \qquad \mathcal{H} = \begin{bmatrix} -k & 0 \\ 0 & -\frac{7}{2} mgl \end{bmatrix}$$

$$\begin{cases} m\ddot{\xi} + k(\xi - e) = 0 \\ \frac{31}{4} ml^2 \ddot{\theta} + \frac{7}{2} mgl\,\theta = 0 \end{cases} \qquad \square$$

II) $\begin{cases} Q = \alpha q^2 \sqrt{p} \\ P = \frac{\sqrt{p}}{q} \end{cases}$

$$[Q, P] = \cancel{2}\alpha \cancel{q} \cancel{\sqrt{p}} \, \frac{1}{\cancel{2}\cancel{q}\cancel{\sqrt{p}}} + \frac{\cancel{\sqrt{p}}}{\cancel{q^2}} \alpha \cancel{q^2} \frac{1}{2\cancel{\sqrt{p}}} = \alpha + \frac{\alpha}{2}$$

$$= \frac{3}{2}\alpha = 1 \qquad \alpha = \frac{2}{3}$$

$\sqrt{p} = q P$ $\qquad \begin{cases} p = q^2 P^2 = \frac{\partial F_2}{\partial q} \\ Q = \frac{2}{3} q^3 P = \frac{\partial F_2}{\partial P} \end{cases}$

$$\Rightarrow \quad F_2(q, P) = \frac{\cancel{2}}{3} q^3 \frac{P^2}{\cancel{2}} + g(q)$$

$$\frac{\partial F_2}{\partial q} = q^2 P^2 + g'(q) = q^2 P^2$$

$$\Rightarrow \quad g'(q) = 0$$

$$\Rightarrow \quad F_2(q, P) = \frac{1}{3} q^3 P^2 + \text{const}$$