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3) $\begin{cases} \dot{x} = -x^3 - ky^3 \\ \dot{y} = x^3 - y^5 \end{cases}$ $(0,0)$ $\boxed{k \geq 0}$

$$\begin{bmatrix} -3x^2 & 3ky^2 \\ 3x^2 & -5y^4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$k>0$ $W = \frac{x^4}{4} + \frac{ky^4}{4}$

$$\dot{W} = (-x^3 - ky^3)x^3 + (x^3 - y^5)ky^3$$

$$= -x^6 - ky^8$$

$$\{\dot{W} = 0\} = \{(0,0)\} \quad A.S.$$

$k=0$ $W = \frac{x^4}{4} + \frac{y^6}{6}$

$$\dot{W} = -x^6 + x^3y^5 - y^{10} \qquad x^3 = t \quad y^5 = s$$

$$= -t^2 + ts - s^2 \leq 0$$

$$\{\dot{W} = 0\} = \{(0,0)\} \quad A.S.$$

2) $x_{n+1} = a \sin x_n$

$\bar{x} = 0$ $\qquad a \cos x \Big|_{x=0} = a$

$|\alpha| < 1$ A.S.

$|\alpha| > 1$ INST

$\alpha = 1$ $\quad f''(x) = -\sin x\big|_{x=0} = 0$

$f'''(x) = -\cos x\big|_{x=0} = -1$ A.S.

$\alpha = -1$ $\quad f''(x) = \sin x\big|_{x=0} = 0$

$f'''(x) = \cos x\big|_{x=0} = 1$

$2 > 0$ A. ST.

$\alpha = \frac{\pi}{2}$ $\quad f(x) = \frac{\pi}{2}\sin x = x$

$x = \frac{\pi}{2}$ A.S. (fare)

5) $\begin{cases} y - a = x \\ a - x^2 = y \end{cases}$ $\quad y = a + x$

$a - x^2 = a + x \quad \begin{matrix} x = 0 \\ x = -1 \end{matrix}$

$(0, a)$ $\quad (-1, a-1)$

$\begin{bmatrix} 0 & 1 \\ -2x & 0 \end{bmatrix}$ $\quad (0,a)\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad \rho = 0$ A.S.

$(-1, a-1)\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ $\quad \lambda^2 = 2 \quad \lambda = \pm\sqrt{2}$ INST.

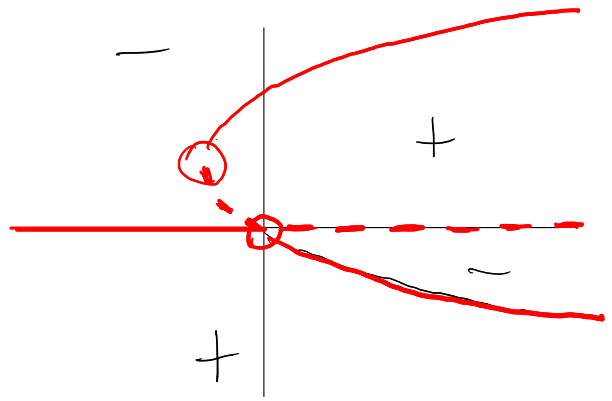
2-cicli:

$$\begin{cases} a - x^2 - a = x \\ a - (y-a)^2 = y \end{cases} \qquad x = 0, \; x = -1$$

$$(y-a)^2 = -(y-a) \quad \begin{cases} y = a \\ y - a = -1 \end{cases} \qquad y = a - 1$$

$$\{(0, a-1), (-1, a)\} \qquad 2\text{-ciclo}$$

4)



1) $\begin{cases} \dot{x} = kx - y \\ \dot{y} = kx + y \end{cases}$

$y = kx = -kx$

$k \neq 0 \Rightarrow x = 0 \Rightarrow (0,0)$

$k = 0 \Rightarrow (\bar{x}, 0)$

$$\begin{bmatrix} k & -1 \\ k & 1 \end{bmatrix} \qquad \tau = k + 1 \qquad \delta = 2k$$

$\Delta = k^2 - 6k + 1$

$\Delta \geq 0 \qquad k \leq 3 - \sqrt{8} \vee k \geq 3 + \sqrt{8}$

$k < 0$ SELLA

$k > 0 \Rightarrow \tau > 0 \Rightarrow$ INST.

$0 < k \leq 3 - \sqrt{8} \vee k \geq 3 + \sqrt{8}$ NODO INST.

$3 - \sqrt{8} < k < 3 + \sqrt{8}$ FUOCO INST.

$k = 0 \quad \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix} \quad \begin{matrix} \lambda = 0 \\ \lambda = 1 \end{matrix}$ $(\bar{x}, 0)$ INST. □