Booklet of Abstracts
Brescia Trento Nonlinear Days
26th May 2017

Regularity of the optimal sets for some (also nonlinear) spectral functionals

Dario Mazzoleni, Università di Pavia

Abstract: First of all we consider the variational problem

$$\min\Big\{\lambda_1(\Omega)+\cdots+\lambda_k(\Omega)\ :\ \Omega\subset\mathbb{R}^d,\ |\Omega|=1\Big\},$$

where the variable is the domain Ω, $|\cdot|$ denotes the Lebesgue measure and the cost functional is the sum of the first k Dirichlet eigenvalues on Ω.

We prove that the optimal sets have C^∞ regular boundary up to a set of zero $\mathcal{H}^{d-1}$−measure. This is strongly related to the regularity of the free boundary $\partial\{|U|>0\}$ of the local minima of the functional

$$H^1_{loc}(\mathbb{R}^d,\mathbb{R}^k)\ni W\mapsto\int|\nabla W|^2+|\{|W|>0\}|,$$

on which we will focus most of our attention. The proof involves techniques from the classical one-phase free boundary problems, monotonicity formulas and a viscosity approach in order to deal with the optimality condition. Moreover a key tool in our proof is a boundary Harnack principle for non-tangentially accessible domains.

Then we will focus on how to extend this regularity result to more general nonlinear functionals of the eigenvalues involving λ_1. This extension is far from trivial and it raises many new technical problems in the proofs, due to the fact that there is no more a direct equivalence with a free boundary problem. We show how to treat this more general case with a double approximation argument.

The talk is mainly based on joint works with Susanna Terracini and Bozhidar Velichkov.

Spiralling asymptotic profiles of competition-diffusion systems

Gianmaria Verzini, Politecnico di Milano

Abstract: In this talk we consider solutions of the competitive elliptic system

$$\begin{cases}-\Delta u_i=-\beta\sum_{j\neq i}a_{ij}u_iu_j & \text{in }\Omega\subset\mathbb{R}^2\\ u_i=g & \text{in }\partial\Omega\end{cases}\qquad i=1,\dots,k,$$

and their asymptotic profiles when $\beta\to+\infty$. We shall focus our attention on the asymmetric case: $a_{ij}\neq a_{ji}$.

This is a joint result with S. Terracini, A. Zilio and A. Salort.

Regularity for a class of functionals with mild phase transition
Paolo Baroni, Università di Parma

Abstract: We present a basic regularity theory for a class of non-autonomous functionals with mild phase transition, whose energy density changes its ellipticity and growth properties according to the point. The functionals we study can be seen as the borderline case of a class of functionals with (p, q) growth introduced by Zhikov in the eighties. This is part of a larger research project in collaboration with M. Colombo and G. Mingione.

An Aleksandrov-type result for circular domains in $\mathbb{C}^2$
Giulio Tralli, Università degli Studi di Roma "La Sapienza"

Abstract: In this talk we consider the problem of characterizing spheres in $\mathbb{C}^2$ by the fact they have constant Levi curvature. The equation of prescribed Levi curvature is a quasilinear degenerate-elliptic equation with an underlying Hrmander structure, and moving plane techniques seem to be not working. We discuss a rigidity result of Jellett-type for a suitable class of real hypersurfaces. As a main application, we provide an Aleksandrov-type result for starshaped domains with circular symmetries. This is a joint work with V. Martino.

Geometric aspects of potential theory
Lorenzo Mazzieri, Università degli Studi di Trento

Abstract: In this talk we show how to apply some geometric tools - such as splitting principles and sphere theorems - to the study of harmonic and p-harmonic functions in exterior domains. Integral identities and monotonicity formulas are obtained and then used to deduce geometric inequalities.

Gauss curvature in the Heisenberg group via Riemannian approximation
Eugenio Vecchi, Università di Bologna

Abstract: The notion of mean and Gauss curvature play a crucial role in the study of differential geometry of smooth Euclidean surfaces embedded in $\mathbb{R}^3$. In the first Heisenberg group $\mathbb{H}$ there is a currently well accepted notion of horizontal mean curvature for smooth Euclidean surfaces $\Sigma \subset \mathbb{H}$, but it is still not well understood what could be a reasonable

candidate for the notion of horizontal Gauss curvature. In this talk I will suggest a possible definition based on the so called Riemannian approximation scheme. The expression for the Gauss curvature arising from this procedure is nonlinear and plays a role in a Heisenberg version of the Gauss-Bonnet theorem for Euclidean C^2−smooth, oriented and compact surfaces with isolated characteristic points in $\mathbb{H}$. These results have been obtained in collaboration with Zoltàn Balogh and Jeremy Tyson.

On optimizers for fractional Sobolev-type inequalities

Sunra Mosconi, Università di Catania

Abstract: We will discuss a conjecture regarding the explicit form of the optimizers for the fractional Sobolev embedding and related inequalities. The optimizers are well known for the classical embedding $W_0^{1,p}(\mathbb{R}^N) \hookrightarrow L^{p^*}(\mathbb{R}^N)$ and in the Hilbertian case $W_0^{s,2}(\mathbb{R}^N) \hookrightarrow L^{2^*_s}(\mathbb{R}^N)$, $s \in]0,1[$, but only partial informations are known in the general case $p \neq 2$, $s \in]0,1[$. We will present two results in support of the conjectured form of the optimizers. The first one is a proof of the precise asymptotic behaviour at infinity of such optimizers. The second one is an energy estimate in Sobolev spaces of lower summability. Both results are compatible with the conjectured form of the optimizers, and are contained in joint works with L. Brasco (Ferrara), N. Marano (Catania) and M. Squassina (Brescia).

Torsional rigidity VS.
homogeneous Sobolev spaces

Lorenzo Brasco, Università degli studi di Ferrara

Abstract: Let Ω be a generic open set of the Euclidean space. We consider the *homogeneous Sobolev space*, defined by the completion of the space of compactly supported smooth functions with respect to the L^p norm of the gradient.

We give a characterization of the continuous (or compact) embedding of this space into $L^q(\Omega)$, in terms of the summability of the so-called *p−torsion function* of Ω. We also introduce a new Hardy-type inequality, which plays an important role in the proofs.

Finally, in the Hilbertian case we give some applications of our Hardy-type inequality to ground state estimates for Schrödinger operators with negative potentials.

The results presented are contained in joint works with Giovanni Franzina (Roma) and Berardo Ruffini (Montpellier).