

# NONLINEAR DAYS IN TURIN

Turin, 21–22 September 2017

## Booklet of Abstracts

Organizers: MARCO SQUASSINA AND SERGIO LANCELOTTI

Timetable	
Thursday 21 September	
9:00 - 9:50	Hoai-Minh Nguyen
10:00 - 10:50	Luca Lussardi
11:00 - 11:30	<b>Coffee break</b>
11:30 - 12:20	Riccardo Molle
<b>Lunch Time</b>	<b>Lunch Time</b>
14:00 - 14:50	Lorenzo Luperi Baglini
15:00 - 15:50	Silvia Cingolani
16:00	<b>Coffee break</b>
Friday 22 September	
9:00 - 9:50	Vieri Benci
10:00 - 10:50	Quoc-Hung Nguyen
11:00 - 11:30	<b>Coffee break</b>
11:30 - 12:20	Berardino Sciunzi
<b>Lunch Time</b>	<b>Lunch Time</b>
14:00 - 14:50	Veronica Felli
15:00 - 15:50	Gianmaria Verzini
16:00	<b>Coffee break</b>

Thursday 21nd September 2017

*Title:* Approximation of BV functions and Sobolev norms by non-local non-convex functionals and the Caffarelli-Kohn-Nirenberg inequalities revisited

**Hoai-Minh Nguyen**, EPFL, Lausanne

**Abstract:** This talk contains two parts. In the first part, I will discuss various results concerning the approximation of the total variation and Sobolev semi-norms by non-local, non-convex functionals. Due to the lack of convexity, the mode of convergence is extremely delicate and many phenomena/pathologies appear. In the second part, I will discuss variants and improvements of the Caffarelli-Kohn-Nirenberg inequalities where the information of the gradient is replaced by the one of the non-local functionals presented in the first part. The first part is based on joint work with Jean Bourgain and Haim Brezis, and the second part is based on joint work with Marco Squassina.

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*Title:* Uniqueness results for non strictly convex variational problems  
**Luca Lussardi**, Politecnico di Torino

**Abstract:** In this seminar I will speak about a recent uniqueness result for a class of non strictly convex variational problems obtained in collaboration with Elvira Mascolo [1]. In particular, we answer the long standing open question which Paolo Marcellini placed in [2]. I will show the technique of the proof which works fine in dimension two and I will discuss the higher dimensional case, studied in collaboration with Giovanni Alberti, which is much more complicated and needs tools from Geometric Measure Theory.

References:

- 1 L. Lussardi and E. Mascolo, *A uniqueness result for a class of non strictly convex variational problems*, accepted on J. Math. Anal. Appl. (2016).
- 2 P. Marcellini, *A relation between existence of minima for nonconvex integrals and uniqueness for non-strictly convex integrals of the calculus of variations*, Mathematical theories of optimization, Genova, 1981, Lecture Notes in Math. 979 (1983), 216–231.

*Title: New results on some Schrödinger equations*  
**Riccardo Molle**, Università di Roma “Tor Vergata”

**Abstract:** In this talk the following model problem is considered

$$(P) \quad -\Delta u + a(x)u = u^p \quad u \in H^1(\mathbb{R}^N), \quad u > 0,$$

where  $N \geq 2$ ,  $p > 1$  and  $p < \frac{N+2}{N-2}$  if  $N \geq 3$ .

In particular, we present some new results concerning the existence of infinitely many positive solutions for (P) and present some perspectives.

*Title: Fixed point methods for singular nonlinear differential equations*

**Lorenzo Luperi Baglini**, University of Wien

**Abstract:** In many situations, especially when dealing with singularities, the notion of function is not sufficient and it needs to be extended. A classical way to do this is to introduce the notion of weak solution; another approach is to use generalized functions. In this talk I want to present some ideas on how to deal with singular differential equations using the spaces of Generalized Smooth Functions (GSF), which are a particular type of functions based on a non-Archimedean extension of the reals. GSF are an extension of the space of Colombeau’s generalized functions and hence, in particular, of the space of distributions. In these spaces nonlinear operations can be performed (almost) unrestrictedly, and many important classical results such as the Banach fixed point theorem and the Picard-Lindelf theorem can be generalized in a rather simple way. We will show how these ideas can be used to introduce a theory of nonlinear generalized ODE and, partially, a theory of nonlinear generalized PDE. Finally, if time allows we will describe the relationships between GSF solutions and weak solutions. This is a joint work with P. Giordano (University of Vienna)

*Title: Sharp asymptotics and nondegeneracy of the groundstate of the logarithmic Choquard equation*

**Silvia Cingolani**, University of Bari

**Abstract:** In my talk I will present some results concerning the asymptotic decay and the nondegeneracy of the unique positive, radially symmetric solution of the logarithmic Choquard equation. This is a joint work with Denis Bonheure (Université libre de Bruxelles) and Jean Van Schaftingen (Université catholique de Louvain).

Friday 22nd September 2017

*Title: La matematica non-archimedeo nel calcolo delle variazioni*  
**Vieri Benci**, Università di Pisa

**Abstract:** Si dice matematica non Archimedeo (NAM) quella parte della matematica che utilizza numeri infiniti ed infinitesimi. Alcuni esempi tratti dal calcolo delle variazioni dimostreranno l'utilità della NAM nella risoluzione di problemi e nella costruzione di modelli matematici.

*Title: Porous medium equation with nonlocal pressure in a bounded domain*

**Quoc-Hung Nguyen**, Scuola Normale Superiore

**Abstract:** We will present a quite general family of nonlinear evolution equations of diffusive type with nonlocal effects. More precisely, they are porous medium equations with a fractional laplacian pressure and they are posed on a bounded space domain. We will discuss existence of weak solutions and different a priori bounds and regularity estimates. This is joint work with Juan Luis Vazquez.

*Title: Monotonicity and rigidity of solutions to some elliptic systems with uniform limits*

**Berardino Sciunzi**, University of Calabria

**Abstract:** Many differences occur when applying the moving plane technique in bounded or in unbounded domains. I will discuss this issue and then I will talk about recent results obtained in collaboration with Alberto Farina and Nicola Soave, for the coupled competing Gross-Pitaevskii system

$$\begin{cases} -\Delta u = u - u^3 - \Lambda uv^2 & \text{in } \mathbb{R}^N \\ -\Delta v = v - v^3 - \Lambda u^2 v & \text{in } \mathbb{R}^N \\ u, v \geq 0 & \text{in } \mathbb{R}^N. \end{cases} \quad \text{with } \Lambda > 0$$

We prove the validity of the Gibbons' conjecture in the case  $\Lambda > 1$  and also we provide Liouville-type theorems in the case  $0 < \Lambda \leq 1$ .

*Title:* On spectral stability of Aharonov-Bohm operators with  
moving poles

**Veronica Felli**, University of Milano Bicocca

**Abstract:** In this talk, I will describe the behavior of the eigenvalues of Aharonov-Bohm operators with one moving pole or two colliding poles. In both cases of poles moving inside the domain and approaching the boundary, the rate of the eigenvalue variation is estimated in terms of the vanishing order of some limit eigenfunction. An accurate blow-up analysis for scaled eigenfunctions will be presented too. The talk is based on results obtained in collaboration with L. Abatangelo (Milano-Bicocca), L. Hillairet (Orléans), C. Léna (Torino), B. Noris (Amiens), M. Nys (Torino).

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*Title:* Normalized bound states for the nonlinear Schrödinger  
equation in bounded domains

**Gianmaria Verzini**, Politecnico di Milano

**Abstract:** We study the existence of solutions having prescribed  $L^2$  mass to some semilinear elliptic equations. When the nonlinearity is critical or supercritical with respect to the Gagliardo-Nirenberg inequality, though Sobolev subcritical, we show that solutions having Morse index bounded from above exist only when the mass is sufficiently small. On the other hand, we provide sufficient conditions for the existence of such solutions.