Booklet of Abstracts
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Geometric inequalities through p-harmonic functionals

Mattia Fogagnolo, Università di Trento

Abstract: We show how the nonlinear level set flow of p-harmonic potentials can be employed in order to derive analytic and geometric inequalities for domains in Euclidean spaces. In particular, we derive a new version of a celebrated inequality known as Minkowski inequality that gives the currently known best Minkowski inequality as a special case. Our proof builds on a monotone quantity for the flow. Time permitting, we will describe the connection with the Inverse Mean Curvature Flow and discuss possible applications to curved settings. The talk is based on joint works with V. Agostiniani, L. Mazzieri and A. Pinamonti.

On the Lane-Emden equation with fully nonlinear degenerate operators

Giulio Galise, Sapienza Università di Roma

Abstract: In this talk we report on some recent results on the existence of viscosity solutions, both in the whole space and bounded convex domain, of a fully nonlinear version of the well known Lane-Emden equation and involving the "truncated Laplacians", a class of highly degenerate operators deriving from geometrical problems about mean partial curvatures. Novelties and unusual phenomena with respect to the semilinear case will be emphasized.

Multi-material transport problems

Annalisa Massaccesi, Università di Verona

Abstract: In this joint works with Andrea Marchese and coauthors, we propose an Eulerian formulation for the multicommodity flow problem, i.e., a general branched transportation problem in which m different goods are moved simultaneously. Moreover, in the atomic case, we prove that the problem can be relaxed in a mass minimization problem in a class of rectifiable currents with coefficients in a group, allowing to introduce a notion of calibration. Finally, I will show how these tools allow the modelling of the oriented mailing problem.

Nonexistence results for semilinear hyperbolic problems on Riemannian manifolds

Dario D. Monticelli, Politecnico di Milano

Abstract: In this talk we will discuss some recent results concerning necessary conditions for the existence of solutions to a class of semilinear hyperbolic problems (where a general weight function depending on spacetime is allowed in front of the power nonlinearity), which are defined on complete noncompact Riemannian manifolds. These conditions extend to the Riemannian setting some well–known nonexistence results for problems involving the wave operator and a power nonlinearity on the whole euclidean space. This is a joint work with F. Punzo (Politecnico di Milano) and M. Squassina (Università Cattolica del Sacro Cuore, Brescia).

Uniqueness of very weak solutions for a fractional filtration equation

Fabio Punzo, Politecnico di Milano

Abstract: We inverstigate existence and uniqueness of distributional, nonnegative, bounded solutions fo the filtration equation in the whole space. We show that a minimal nonnnegative bounded solution exists and that any other solution must conicide with it. The results have been recently obtained jointly with Gabriele Grillo and Matteo Muratori.

Asymptotics for a heat equation with critical exponential nonlinearity

Federica Sani, Università di Milano

Abstract: We consider the Cauchy problem for a 2-space dimensional heat equation with exponential nonlinearity. More precisely, we consider initial data in $H^1(\mathbb{R}^2)$, and a square-exponential nonlinearity, which is critical in the energy space $H^1(\mathbb{R}^2)$ in view of the Trudinger-Moser inequality. The idea is to extend the classical potential well argument of Payne and Sattinger to study the asymptotics of solutions below the ground state energy level. We show that, for energies below the ground state level, the dichotomy between blow-up and global existence is determined by the sign of a suitable family of functionals, and global solutions dissipate. This is a joint work with Michinori Ishiwata, Bernhard Ruf, and Elide Terraneo.

The nodal set of solutions to sublinear and singular equations

Nicola Soave, Politecnico di Milano

Abstract: We are concerned with the nodal set of solutions to sublinear equations of the form

$$-\Delta u = \lambda_+ \left(u^+\right)^{q-1} - \lambda_- \left(u^-\right)^{q-1} \quad \text{in } B_1$$

where $\lambda_+, \lambda_- > 0$, $q \in (0,2)$, $B_1 = B_1(0)$ is the unit ball in $\mathbb{R}^N$, $N \geq 2$, and $u^+ := \max\{u, 0\}$, $u^- := \max\{-u, 0\}$ are the positive and the negative part of u, respectively. This class includes equations with sublinear equations ($1 < q < 2$), the unstable two-phase membrane problem ($q = 1$), and singular equations ($0 < q < 1$). In any case, the right hand side is not locally Lipschitz continuous as function of u.

In this talk we present results regarding: (a) the validity of the unique continuation principle; (b) the finiteness of the vanishing order at every point and the complete characterization of the order spectrum; (c) a weak non-degeneracy property; (d) the partial regularity of the nodal set of any solution: the nodal set is a locally finite collection of regular codimension one manifolds up to a residual singular set having Hausdorff dimension at most $N-2$ (locally finite when $N = 2$) and a partial stratification theorem.

Ultimately, the main features of the nodal set are strictly related with those of the solutions to linear (or superlinear) equations, with two remarkable differences. First of all, the admissible vanishing orders can not exceed the critical value $2/(2-q)$. At threshold, we find a multiplicity of homogeneous solutions, yielding the *non-validity* of any estimate of the $(N-1)$-dimensional measure of the nodal set of a solution in terms of the vanishing order.

For the singular case, we also discuss the regularity of the solutions.

The proofs are based on Weiss-type monotonicity formulæ, blow-up arguments and the classification of homogenous solutions.

The talk is based on joint works with Tobias Weth and Susanna Terracini.

On the logarithmic epiperimetric inequality

Bozhidar Velichkov, Università di Napoli Federico II and Université Grenoble Alpes

Abstract: This talk is dedicted to some recent advances on the regularity of the free boundaries arising in variational minimization problems. In particular, we will present a new variational approach, based on the so-called *logarithmic epiperimetric inequalities*, which is very general and was recently applied to several different free boundary problems. Most of the talk will be carried out in the context of the classical obstacle problem

$$\min\left\{\int_{B_1}(|\nabla u|^2+u)\,dx : u\in H^1(B_1),\ u\geq 0 \text{ in } B_1,\ u \text{ is prescribed on } \partial B_1\right\}. \tag{1}$$

Given a solution u of (1), the free boundary $\partial\{u_1>0\}\cap B_1$ can be decomposed into a regular part, $Reg(\partial\{u>0\})$, and a singular part, $Sing(\partial\{u>0\})$, where

- $Reg(\partial\{u>0\})$ are the points at which the set $\{u>0\}$ has Lebesgue density $1/2$;
- $Sing(\partial\{u>0\})$ are the points at which the Lebesgue density of the set $\{u>0\}$ vanishes.

Caffarelli proved that $Reg(\partial\{u>0\})$ is locally the graph of a smooth function, while the structure of the singular set $Sing(\partial\{u>0\})$ is more complex and, in general, wilder. In this talk, we will prove a *logarithmic epiperimetric inequality* for the Weiss energy, from which we will deduce that $Sing(\partial\{u>0\})$ is contained into the countable union of $C^{1,log}$-regular manifolds. The results and the techniques that I will present are from the recent papers [SV], [CSV] and [ESV].

References

[CSV] M. Colombo, L. Spolaor, B. Velichkov. A logarithmic epiperimetric inequality for the obstacle problem. Geom. Funct. Anal. (2018).

[ESV] M. Engelstein, L. Spolaor, B. Velichkov. Uniqueness of the blow-up at isolated singularities for the Alt-Caffarelli functional. ArXiv 2018.

[SV] L. Spolaor, B. Velichkov. An epiperimetric inequality for the regularity of some free boundary problems: the 2-dimensional case. Comm. Pure. Appl. Math. (2018).