Booklet of Abstracts
Brescia-Trento Nonlinear Day
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The prescribed mean curvature equation in the Lorentz-Minkowski space: new regularity results and open problems

Alessandro Iacopetti *(Università di Torino)*

In this talk we present some recent results concerning the existence and the regularity of weak solutions for the prescribed mean curvature equation in the Lorentz-Minkowski space for entire spacelike hypersurfaces, when the mean curvature belongs to L^p.

In the first part of the talk we will show a new gradient estimate for entire smooth solutions of the prescribed mean curvature equation. Then we will prove that, if p is strictly greater than the dimension N, then the unique minimizer of the Born-Infeld energy, which is a priori only Lipschitz continuous, is actually a strictly spacelike weak solution of the equation and it is of class $W^{2,p}_{loc}$. Finally we will discuss some open problems concerning the case of bounded domains.

These results are collected in a series of joint works with Prof. D. Bonheure (Université Libre de Bruxelles) and a work in progress with Prof. X. Zhong (University of Helsinki), Dr. A. Haarala (University of Helsinki).

Missing Nehari & Pohozaev: normalized solutions for a logarithmic-type fractional equation

Marco Gallo *(Università Cattolica del Sacro Cuore, Brescia)*

Aim of this talk is the research of solutions, with *prescribed mass* $\int_{\mathbb{R}^N} u^2 = m$, to a class of fractional Schrödinger equations

$$(-\Delta)^s u + \omega u = f(u) \quad \text{in } \mathbb{R}^N;$$

due to the mass constraint, the frequency $\omega > 0$ is part of the unknowns.

The peculiarity of the results rely on the general assumptions made on f: indeed, we avoid monotonicity and homogeneity conditions, and this makes the research more involved. Moreover, we allow f to be negatively *sublinear* in the origin: this leads both to examples of power type $f(u) \sim u^p - u^q$ and logarithmic type $f(u) \sim u \log(u^2)$.

The methods involved will be based on a Lagrangian formulation of the problem, as well as on an approximation argument: in particular, we will manage to obtain the result even if two famous tools - the Nehari and the Pohozaev identities - are not part of the crew.

The results are based on joint works with S. Cingolani, K. Tanaka and J. Schino.

Bijections between isoperimetric sets, prescribed curvature sets, and p-Cheeger sets

Giorgio Saracco *(Università di Trento)*

During this talk we will consider three different minimization problems among subsets of a planar, open set Ω, satisfying some weak regular assumptions.

The isoperimetric problem in Ω amounts to determining the sets minimizing

$$\mathcal{J}(V) := \inf\{\, P(E) \,:\, E \subset \Omega,\, |E| = V \,\},$$

for $V \leq |\Omega|$. Denoting by R the radius of the greatest ball contained in Ω, the characterization of minimizers is trivial for $V \leq \pi R^2$.

The prescribed curvature problem amounts to determining the sets minimizing

$$\mathcal{F}(\kappa) := \inf\{\, P(E) - \kappa|E| \,:\, E \subset \Omega,\, |E| \geq \pi\kappa^{-2} \},$$

for curvatures $\kappa \geq R^{-1}$.

Finally, the p-Cheeger problem amounts to determining those minimizing

$$H(p) := \inf\left\{ \frac{P(E)}{|E|^p} \,:\, E \subset \Omega,\, |E| > 0 \right\},$$

for powers $p \geq 1/2$.

Under some geometrical assumptions on Ω, we prove that there exists a bijection $\mathfrak{K}$ between volumes $V \geq \pi R^2$ and curvatures $\kappa \geq R^{-1}$, in such a way that a set minimizes $\mathcal{J}(V)$ if and only if it minimizes $\mathcal{F}(\mathfrak{K}(V))$. An analog result can be shown between powers $1/2 \leq p \leq 1$ and suitable intervals of volumes and curvatures. As a byproduct, we infer some convexity properties on the profile $\mathcal{J}$, and some fine regularity properties on the contact surface of minimizers.

This is based on joint works with Caroccia, Leonardi, Neumayer, and Pratelli.

On the Jacobian determinant of BV functions and its application to Ginzburg-Landau type vortices

Riccardo Scala *(Università di Siena)*

I will discuss a model which, by Gamma convergence, shows the appearance of vortices of the Ginzburg-Landau type starting from a Munford-Shah type functional. This 2D-model is inspired by the dislocation mechanics in the antiplane setting. To approach the problem, a notion of distributional determinant for the gradient of a vector-valued BV map is necessary.

This is a joint work with Lucia De Luca and Nicolas Van Goethem.

An elementary proof of existence and uniqueness for Euler and Vlasov–Poisson flows in localized Yudovich spaces

Giorgio Stefani *(SISSA, Trieste)*

We prove existence and uniqueness of solutions for the 2-dimensional Euler equations and the d-dimensional Vlasov–Poisson equations in localized Yudovich spaces. Our approach relies on elementary real-variable Lagrangian techniques, which apply not only to Biot–Savart and Riesz kernels, but also to more general kernels satisfying some natural structural assumptions.

This is a joint work with Gianluca Crippa, Marco Inversi, and Chiara Saffirio.

Boundedness of Riesz transforms on $RCD(K,\infty)$ spaces

Luca Tamanini *(Università Cattolica del Sacro Cuore, Brescia)*

A well-known result by Bakry states, on any weighted Riemannian manifold with Ricci curvature bounded from below, the boundedness of the Riesz transform w.r.t. the L^p norm, for any $1 < p < \infty$, with dimension-free constants.

After revising the original proof and recent generalizations, we suggest a different approach, relying on a suitable choice of Bellman function. This approach can be extended to metric measure spaces with Riemannian Ricci curvature bounded from below, namely RCD spaces, without any restriction on their dimension. This large class of spaces include e.g. that of Hilbert spaces endowed with a log-concave probability measure.

As an application, we extend the range of validity of the Lusin-type approximation of Sobolev by Lipschitz functions, previously obtained by L. Ambrosio, E. Bruè and D. Trevisan in the quadratic case, i.e. $p = 2$.

(Joint work with A. Carbonaro and D. Trevisan)