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Mathematical modelling of initially stressed materials

A soft solid is said to be initially stressed if it is subjected to a state of internal stress in its unloaded reference configuration. In physical terms, its stored elastic energy may not vanish in the absence of an elastic deformation, being also dependent on the spatial distribution of the underlying material inhomogeneities. Developing a sound mathematical framework to model initially stressed solids in nonlinear elasticity is key for many applications in engineering and biology. In this talk, we show the links between the existence of elastic minimizers and the constitutive restrictions for initially stressed materials subjected to finite deformations. In particular, we consider a subclass of constitutive responses in which the strain energy density is taken as a scalar-valued function of both the deformation gradient and the initial stress tensor. Assuming that the constitutive response depends on the choice of the reference configuration only through the initial stress tensor, under given conditions we prove the local existence of a relaxed state given by an implicit tensor function of the initial stress distribution. If time will allow, we will show some applications of this theory. We will present the stability analysis of an incompressible sphere subject to residual stress and the derivation of the Föppl–von Kármán equations of an initially stressed thin plate as the asymptotic limit of the three dimensional problem.

References:

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