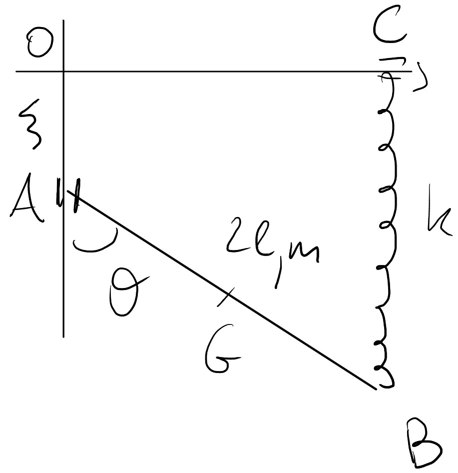
MA 26/6/26

$$\lambda = \frac{mg}{kl}$$

I)



$\xi \in \mathbb{R}$

$\theta \in [0, 2\pi]$

$(G-A) = l\sin\theta, -l\cos\theta$

$(G-O) = l\sin\theta, -l\cos\theta - \xi$

1) $B-O = 2l\sin\theta, -2l\cos\theta - \xi$

$B-C = 0, -2l\cos\theta - \xi$

$$U = mgl\cos\theta + mg\xi - \frac{1}{2}k(\xi + 2l\cos\theta)^2$$

$$\frac{\partial U}{\partial \xi} = mg - k(\xi + 2l\cos\theta) \qquad = 0$$

$$\frac{\partial U}{\partial \theta} = -mgl\sin\theta + k(\xi + 2l\cos\theta)2l\sin\theta = 0$$

- $\sin\theta = 0 \qquad \theta = 0 \qquad mg - k(\xi + 2l) \qquad = 0$

$$k\xi = mg - 2kl$$

$$\xi = \frac{mg}{k} - 2l = \lambda l - 2l = (\lambda - 2)l$$

$$\theta = \pi \qquad mg - k(\xi - 2l) \qquad = 0$$

$$\xi = \lambda l + 2l = (\lambda + 2)l$$

$$-mg\cancel{\ell} + k(\xi + 2\ell\cos\vartheta)2\cancel{\ell} = 0$$

$$k(\xi + 2\ell\cos\vartheta) = \frac{mg}{2}$$

$$mg - \frac{mg}{2} \qquad = 0 \qquad \text{MAI}$$

$$P_1\big((\lambda-2)\ell, 0\big) \qquad P_2\big((\lambda+2)\ell, \pi\big)$$

2) $$\partial H = \begin{bmatrix} -k & 2k\ell\sin\vartheta \\ 2k\ell\sin\vartheta & -mg\ell\cos\vartheta + 2k\ell\xi\cos\vartheta + 4k\ell^2(\cos^2\vartheta - \sin^2\vartheta) \end{bmatrix}$$

$$H(P_1) = \begin{bmatrix} -k & 0 \\ 0 & -mg\ell + 2k\ell(\lambda-2)\ell + 4k\ell^2 \end{bmatrix}$$

$$-mg\ell + 2k\ell^2\lambda - \cancel{4k\ell^2} + \cancel{4k\ell^2}$$

$$= k\ell^2(-\lambda + 2\lambda) = \lambda k\ell^2 > 0$$

INST

$$\mathcal{H}(P_2) = \begin{bmatrix} -k & 0 \\ 0 & +mgl - 2kl(\lambda+2)l + 4kl^2 \end{bmatrix}$$

$$mgl - 2kl^2\lambda - \cancel{4kl^2} + \cancel{4kl^2}$$

$$kl^2(\lambda - 2\lambda) < 0 \qquad P_2 \text{ STABILE}$$

3)
$$K = \frac{1}{2} m v_G^2 + \frac{1}{2} \omega \cdot J_G \omega$$

$$v_G = l\cos\theta\,\dot{\theta},\ l\sin\theta\,\dot{\theta} - \dot{\xi}$$

$$|v_G|^2 = l^2\dot{\theta}^2 + \dot{\xi}^2 - 2l\dot{\xi}\dot{\theta}\sin\theta$$

$$J_G^{33} = \frac{1}{12} m (2l)^2 = \frac{ml^2}{3}$$

$$K = \frac{1}{2} m \dot{\xi}^2 + \frac{1}{2} m l^2 \dot{\theta}^2 - ml\dot{\xi}\dot{\theta}\sin\theta + \frac{1}{2}\frac{ml^2}{3}\dot{\theta}^2$$

$$= \frac{1}{2}\left[m\dot{\xi}^2 + \frac{4}{3} ml^2\dot{\theta}^2 - 2ml\dot{\xi}\dot{\theta}\sin\theta\right]$$

$$\mathbb{K} = \begin{bmatrix} m & -ml\sin\theta \\ -ml\sin\theta & \frac{4}{3} ml^2 \end{bmatrix}$$

$$\mathbb{K}(P_2) = \begin{bmatrix} m & 0 \\ 0 & \frac{4}{3} m\ell^2 \end{bmatrix} \qquad \mathcal{H}(P_2) = \begin{bmatrix} -k & 0 \\ 0 & -\lambda k\ell^2 \end{bmatrix}$$

$$\tilde{\mathcal{L}} = \frac{1}{2} \begin{bmatrix} \dot{\xi} & \dot{\theta} \end{bmatrix} \mathbb{K}(P_2) \begin{bmatrix} \dot{\xi} \\ \dot{\theta} \end{bmatrix} - \begin{bmatrix} \xi-(\lambda+2)\ell & \theta-\pi \end{bmatrix} \mathcal{H} \begin{bmatrix} \cdots \\ \cdots \end{bmatrix}$$

eq. diff. linearizzate

$$\begin{cases} m\ddot{\xi} + k(\xi - (\lambda+2)\ell) = 0 \\ \frac{4}{3} m\ell^2 \ddot{\theta} + \lambda k\ell^2 (\theta - \pi) = 0 \end{cases}$$

II)

$$J_G^Q = \begin{bmatrix} \frac{m\ell^2}{3} & & \\ & \frac{m\ell^2}{3} & \\ & & \frac{2}{3} m\ell^2 \end{bmatrix}$$

$$J_G^D = \begin{bmatrix} \frac{m\ell^2}{4} & & \\ & \frac{m\ell^2}{4} & \\ & & \frac{m\ell^2}{2} \end{bmatrix}$$

$$J_O^Q = \begin{bmatrix} 4\frac{m\ell^2}{3} & & \\ & 4\frac{m\ell^2}{3} & \\ & & \frac{8}{3} m\ell^2 \end{bmatrix}$$

$$J_0^{D} = \begin{bmatrix} \frac{5}{4} m\ell^2 & & \\ & \frac{5}{4} m\ell^2 & \\ & & \frac{5}{2} m\ell^2 \end{bmatrix}$$

I prod. d'inerzia si annullano!

$$\frac{8}{3} + \frac{5}{2} = \frac{16+15}{6} = \frac{31}{6}$$

$$J_0^{TOT} = \begin{bmatrix} \frac{31}{6} m\ell^2 & 0 & 0 \\ 0 & \frac{31}{6} m\ell^2 & 0 \\ 0 & 0 & \frac{31}{3} m\ell^2 \end{bmatrix}$$

$\square$