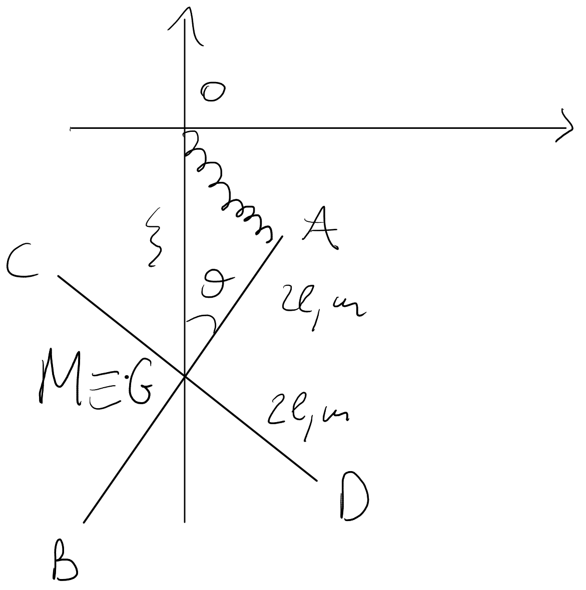
MA 17/7/26



$$(A-O) = (A-G) + (G-O)$$
$$= (l\sin\theta,\ l\cos\theta) + (0, -\xi)$$
$$= (l\sin\theta,\ l\cos\theta - \xi)$$
$$|A-O|^2 = l^2 + \xi^2 - 2l\xi\cos\theta$$

$$U = 2mg\xi - \frac{1}{2}k\xi^2 + kl\xi\cos\theta$$

$$\frac{\partial U}{\partial \xi} = 2mg - k\xi + kl\cos\theta = 0$$

$$\frac{\partial U}{\partial \theta} = -kl\xi\sin\theta = 0$$

$$\xi \sin\theta = 0 \quad \begin{cases} \xi = 0 \\ \theta = 0, \pi \end{cases}$$

$\xi = 0 \quad \Rightarrow \quad \cos\theta = -2\lambda \qquad \alpha = \arccos 2\lambda$

$\theta = \pi + \alpha, \quad \theta = 2\pi - \alpha \qquad \lambda < \frac{1}{2}$

$\theta = 0 \qquad k\xi = 2mg + kl \qquad \xi = l + 2\lambda l$

$\theta = \pi \qquad k\xi = 2mg - kl \qquad \xi = -l + 2\lambda l$

$$\mathcal{H} = \begin{bmatrix} -k & -kl\sin\vartheta \\ -kl\sin\vartheta & -kl\xi\cos\vartheta \end{bmatrix}$$

$$\mathcal{H}(0, \pi+\alpha) = \begin{bmatrix} -k & kl\sin\alpha \\ kl\sin\alpha & 0 \end{bmatrix}$$

$$\det \mathcal{H} = -(kl\sin\alpha)^2 < 0 \qquad \text{INST}$$

$\mathcal{H}(0, 2\pi-\alpha)$ idem INST

$$\mathcal{H}((1+2\lambda)l, 0) = \begin{bmatrix} -k & 0 \\ 0 & -kl^2(1+2\lambda) \end{bmatrix}$$

STAB.

$$\mathcal{H}((2\lambda-1)l, \pi) = \begin{bmatrix} -k & 0 \\ 0 & kl^2(2\lambda-1) \end{bmatrix}$$

STAB se $2\lambda - 1 < 0 \Rightarrow \lambda < \frac{1}{2}$

INST. se $\lambda > \frac{1}{2}$

$$K = \frac{1}{2}(2m)v_G^2 + \frac{1}{2} J_G^{33}\dot{\vartheta}^2$$

$$J_G^{33} = \frac{2}{3}ml^2 \qquad v_G^2 = \dot{\xi}^2$$

$$K = m\dot{\xi}^2 + \frac{1}{3}ml^2\dot{\theta}^2$$

$$\mathcal{L} = m\dot{\xi}^2 + \frac{1}{3}ml^2\dot{\theta}^2 + 2mg\xi - \frac{1}{2}k\xi^2 + kl\xi\cos\theta$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\xi}} = 2m\dot{\xi} \qquad \frac{\partial\mathcal{L}}{\partial\xi} = 2mg - k\xi + kl\cos\theta$$

$$2m\ddot{\xi} = 2mg - k\xi + kl\cos\theta$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = \frac{2}{3}ml^2\dot{\theta} \qquad \frac{\partial\mathcal{L}}{\partial\theta} = -kl\xi\sin\theta$$

$$\frac{2}{3}ml^2\ddot{\theta} = -kl\xi\sin\theta$$

$$P = ((1+2\lambda)l, 0) \qquad \lambda = 1 \qquad P = (3l, 0)$$

$$\tilde{\mathcal{L}} = m\dot{\xi}^2 + \frac{1}{3}ml^2\dot{\theta}^2 - \frac{1}{2}k(\xi - 3l)^2 - \frac{3}{2}kl^2\theta^2$$

$$\begin{cases} 2m\ddot{\xi} = -k(\xi - 3l) \\ \frac{2}{3}m\cancel{l^2}\ddot{\theta} = -3k\cancel{l^2}\theta \end{cases} \qquad \begin{cases} \ddot{\xi} + \frac{k}{2m}(\xi - 3l) = 0 \\ \ddot{\theta} + \frac{9}{2}\frac{k}{m}\theta = 0 \end{cases}$$

II) $J_G^{disco} = \begin{bmatrix} \frac{ml^2}{4} & 0 & 0 \\ 0 & \frac{ml^2}{4} & 0 \\ 0 & 0 & \frac{ml^2}{2} \end{bmatrix}$

$d = (G-O) = (-l, l, 0)$

$\|d\|^2 = 2l^2$

$d \otimes d = \begin{bmatrix} l^2 & l^2 & 0 \\ l^2 & l^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$J_O^{disco} = \begin{bmatrix} \frac{ml^2}{4} + m(2l^2 - l^2) & ml^2 & 0 \\ ml^2 & \frac{ml^2}{4} + m(2l^2 - l^2) & 0 \\ 0 & 0 & '' \end{bmatrix}$$

$$= ml^2 \begin{bmatrix} \frac{5}{4} & 1 & 0 \\ 1 & \frac{5}{4} & 0 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$$

$$J_G^{quad} = \begin{bmatrix} \frac{ml^2}{3} & 0 & 0 \\ 0 & \frac{ml^2}{3} & 0 \\ 0 & 0 & \frac{2}{3}ml^2 \end{bmatrix}$$

$d = (l, l, 0)$

$$J_O^{quad} = \begin{bmatrix} \frac{ml^2}{3} + ml^2 & -ml^2 & 0 \\ -ml^2 & \frac{4}{3}ml^2 & 0 \\ 0 & 0 & \frac{8}{3}ml^2 \end{bmatrix} = ml^2 \begin{bmatrix} 4/3 & -1 & 0 \\ -1 & 4/3 & 0 \\ 0 & 0 & \frac{8}{3} \end{bmatrix}$$

$$J_0^{TOT} = J_0^{ding} + 2 J_0^{quad} =$$

$$\frac{5}{4} + \frac{8}{3} = \frac{15+32}{12}$$

$$= m\ell^2 \begin{bmatrix} \frac{47}{12} & -1 & 0 \\ -1 & \frac{47}{12} & 0 \\ 0 & 0 & \frac{47}{6} \end{bmatrix} .$$

$\square$