SD 17/7/26

1) $kx + ky = 0 \qquad y + ky = 0$

$kx - y = 0 \qquad y = kx$

$k \neq 0,\ k \neq -1 \qquad y = 0 \quad x = 0 \qquad (0,0)$

$k = -1 \quad (-\bar{y}, \bar{y}) \qquad k = 0 \quad (\bar{x}, 0)$

$$\begin{bmatrix} k & k \\ k & -1 \end{bmatrix} \qquad \tau = k-1 \qquad \delta = -k-k^2$$

$-k(k+1) < 0 \quad k(k+1) > 0 \qquad k < -1 \vee k > 0$ SELLA

$-1 < k < 0 \qquad \tau^2 - 4\delta = k^2 - 2k + 1 + 4k + 4k^2$

$= 5k^2 + 2k + 1 > 0$

$\tau < 0 \quad k - 1 < 0 \quad k < 1$

$\Rightarrow -1 < k < 0$ NODO ST.

$k = 0,\ k = -1$ casi degeneri

$k = 0 \quad \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \qquad (\bar{x}, 0)$ S. STAB

$k = -1 \quad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \qquad (1+\lambda)^2 = 1$

$1+\lambda=\pm 1 \qquad \lambda = \begin{cases} 0 \\ -2 \end{cases} \qquad (-\bar{y}, \bar{y})$ S. STAB.

2) $f(x) = \alpha \operatorname{tg} x$

$f'(x) = \alpha(1+\operatorname{tg}^2 x)$

$f''(x) = \alpha\, 2\operatorname{tg} x(1+\operatorname{tg}^2 x)$

$f'''(x) = 2\alpha\left(1+\operatorname{tg}^2 x + 3\operatorname{tg}^2 x(1+\operatorname{tg}^2 x)\right)$

$f'(0) = \alpha \qquad -1<\alpha<1 \quad 0$ A.S.

$\alpha<-1 \vee \alpha>1 \quad 0$ INST.

$\alpha=1 \qquad f'(0)=1 \qquad f''(0)=0 \qquad f'''(0)=2>0$

0 INST.

$\alpha=-1 \qquad f'(0)=-1 \qquad f''(0)=0 \qquad f'''(0)=-2$

$2f'''(0) - 3f''^2(0) = -4 < 0$ INST.

3) $\begin{cases} \dot{x} = -x^5 - ky \\ \dot{y} = +x^3 - y^3 \end{cases} \qquad \begin{bmatrix} 0 & -k \\ 0 & 0 \end{bmatrix}$ $k<0$ INST. $\circledast$ per CETAEV (difficile, vedi in fondo)

$k>0 \qquad W = \dfrac{x^4}{4} + k\dfrac{y^2}{2}$

$\dot{W} = -x^8 - \cancel{kx^3y} + \cancel{kx^3y} - ky^4 \leq 0$

$\{\dot{W} = 0\} = (0,0) \Rightarrow$ A.S.

$k = 0 \quad W = \frac{x^2}{2} + \frac{y^n}{n}$

$$\dot{W} = -x^6 + x^3 y^{n-1} - y^{2+n}$$

$2+n = 2(n-1) \qquad 2+n = 2n-2$

$n = 4$

$$W = \frac{x^2}{2} + \frac{y^4}{4}$$

$$\dot{W} = -x^6 + x^3 y^3 - y^6 \leq 0$$

FALSO

QUADRATO

$\{\dot{W} = 0\} = (0,0) \Rightarrow$ A.S.

4) $\begin{cases} x_{k+1} = a y_k \\ y_{k+1} = 1 - x_k \end{cases} \qquad \begin{matrix} a y = x \\ 1 - x = y \end{matrix}$

$a(1-x) = x \qquad (a+1)x = a \qquad x = \frac{a}{a+1}$

$y = \frac{1}{a+1} \qquad a \neq -1$

$\begin{bmatrix} 0 & a \\ -1 & 0 \end{bmatrix} \qquad \begin{matrix} \tau = 0 \\ \delta = a \end{matrix}$

$0 < 1 + a < 2$

$-1 < a < 1 \qquad \left(\frac{a}{a+1}, \frac{1}{a+1}\right)$ A. S.

$a < -1 \vee a > 1 \qquad$ INST.

$a = 1 \qquad \left(\frac{1}{2}, \frac{1}{2}\right)$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \lambda^2 + 1 = 0 \qquad \lambda = \pm i$$

λ distinti, sist. lineare

$\Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right)$ S. STAB.

$$a = -1 \qquad \begin{cases} x_{n+1} = -y_n \\ y_{n+1} = 1 - x_n \end{cases}$$

$$f(f(x,y)) = (-1+x, 1+y)$$

$$\begin{cases} -1 + x = x \\ 1 + y = y \end{cases} \qquad \text{NO 2-cicli.}$$

$\square$

(*) Instabilità di $(0,0)$ per $\begin{cases} \dot{x} = -x^5 - ky \\ \dot{y} = x^3 - y^3 \end{cases}$ $k<0$

Poniamo $k = -c$, $c > 0$ $\begin{cases} \dot{x} = -x^5 + cy \\ \dot{y} = x^3 - y^3 \end{cases}$ $c > 0$

$W = xy$ $A = \text{I quadrante}$ $\Rightarrow$ $(0,0) \in \partial A$

$W > 0$ su A

$$\dot{W} = -x^5 y + cy^2 + x^4 - xy^3$$

$$= \underbrace{x^4(1-xy)}_{\substack{>0 \\ \text{su } A\cap U}} + \underbrace{y^2(c-xy)}_{\substack{>0 \\ \text{su } A\cap U}}$$

se U intorno di $(0,0)$ abbastanza piccolo

$\Rightarrow \dot{W} > 0$

$\Rightarrow (0,0)$ instabile per Četaev. $\square$