

Dynamical Systems in Finance - Written test - January 14, 2022

1. The dynamics of a price in a Cobweb model is described by the equations

$$D_h = -\frac{1}{2}p_h + 1, \quad S_{h+1} = p_h + \frac{1}{2}$$

where D is the demand and S the supply.

Find the equilibrium position and discuss its stability. Moreover find the price at time $h = 4$ supposing that $p_0 = 1$.

2. The dynamics of price variations $P_1, P_2 \in \mathbb{R}$ of two goods is described by the nonlinear system

$$\begin{cases} \dot{P}_1 = P_1(1 - P_2) \\ \dot{P}_2 = P_2(P_1 + k). \end{cases}$$

where $k > 0$. Find the equilibrium positions and discuss their stability.

3. A gambler plays a game where he/she can win 1€ with probability $2/3$. What is the probability of the gambler's ruin if the target is to reach 5€ and he/she starts from 2€?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{2}{3}Y + G - 2 \\ \dot{G} = -G - \int_0^t Y(\tau)d\tau. \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Consider the discrete map

$$x_{h+1} = x_h - \frac{8}{x_h}.$$

Find the equilibria (if any) and discuss their stability. Moreover, find the 2-cycle.

What happens if the number 8 is replaced by a parameter $d > 0$?