

Dynamical Systems in Finance - Written test - February 4, 2022

1. Find the equilibrium of the linear discrete dynamical system

$$\begin{cases} x_{h+1} = x_h + 2y_h - 1 \\ y_{h+1} = 3x_h + 7y_h - 3 \end{cases}$$

and discuss its stability.

2. The dynamics of price variations $P_1, P_2 \in \mathbb{R}$ of two goods is described by the nonlinear system

$$\begin{cases} \dot{P}_1 = P_1(kP_2 - 2) \\ \dot{P}_2 = -P_2(2P_1 + 1). \end{cases}$$

where $k \neq 0$. Find the equilibrium positions and discuss their stability.

3. A gambler plays a game where he/she can win 1€ with probability 1/3. What is the probability of the gambler's ruin if the target is to reach 5€ and he/she starts from 2€?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{1}{2}Y + G - 1 \\ \dot{G} = -2G - \int_0^t Y(\tau)d\tau. \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Consider the discrete map

$$x_{h+1} = x_h + 1 - \sqrt{x_h}$$

defined for $x_0 > 0$. Find the equilibria (if any) and discuss their stability.