

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **December 19th, 2022**

1. The dynamics of a price in a Goodwin model is described by the equation

$$p_{h+2} + \frac{4}{3}p_{h+1} - \frac{1}{4}p_h = 10.$$

Find the equilibrium position and discuss its stability. Moreover, write all the explicit solutions.

2. What is the probability of a gambler's ruin if the target is to win 150€ and he/she starts from 20€? Assume that the game is fair.

3. Discuss the stability of the origin in the system of ODEs

$$\begin{cases} \dot{x} = y \\ \dot{y} = -2x - 3y \\ \dot{z} = -\alpha z \end{cases}$$

for $\alpha \in \mathbb{R}$. Are there any other equilibrium positions?

4. A Phillips' stabilization model with mixed policy is of the form

$$\begin{cases} \dot{Y} = -3Y + G - 2 \\ \dot{G} = -G - 2Y - 2\dot{Y} \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.s.

$$\begin{cases} x_{h+1} = x_h - y_h \\ y_{h+1} = \alpha x_h - y_h^2 \end{cases}$$

for $\alpha \in \mathbb{R}$.