

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **January 13th, 2023**

1. The dynamics of a price in a Goodwin model is described by the equation

$$p_{h+2} + \frac{2}{3}p_{h+1} - \frac{1}{3}p_h = 4.$$

Find the equilibrium position and discuss its stability. Moreover, write all the explicit solutions.

2. What is the probability of a gambler's ruin if the target is to win 4€ and he/she starts from 2€? Assume that the gambler has a 60% chance of winning.

3. Discuss the stability of the origin in the system of ODEs

$$\begin{cases} \dot{x} = \alpha x \\ \dot{y} = -2y - 3z \\ \dot{z} = 2y + z \end{cases}$$

for $\alpha \in \mathbb{R}$. Are there any other equilibrium positions?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{1}{2}Y + G - 3 \\ \dot{G} = -2G - 2 \int_0^t Y(\tau) d\tau. \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h^2 + \alpha x_h$$

for $\alpha \in \mathbb{R}$.