

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **June 30th, 2023**

1. The dynamics of a price in a Goodwin model with regressive expectation is described by the equation

$$2p_{h+2} + p_{h+1} + p_h = 3.$$

Find the equilibrium position and discuss its stability.

2. A gambler wins 1€ with probability $q = 1/3$ and loses 1€ with probability $1 - q = 2/3$. If he/she wants to reach a target of 6€ starting from 3€, what is the probability of getting ruined?

What should be the value of q in order to lower the probability of getting ruined below 50%?

3. Discuss the stability of the origin in the system of ODEs

$$\begin{cases} \dot{x} = -\alpha x \\ \dot{y} = -\alpha y - z \\ \dot{z} = x - z \end{cases}$$

for $\alpha \in \mathbb{R}$. Are there any other equilibrium positions?

4. A Phillips' stabilization model with proportional policy is of the form

$$\begin{cases} \dot{Y} = -3Y + G - 3 \\ \dot{G} = -G - \frac{1}{2}Y \end{cases}$$

Write the equilibrium position and discuss its stability.