

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **December 11th, 2023**

1. The dynamics of a price in a Goodwin model is described by the equation

$$3p_{h+2} + (\alpha + 1)p_{h+1} - \alpha p_h = 4, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case $\alpha = 1$ discuss the stability and write the explicit solutions.

2. A gambler starts from 1€ and has a target of winning 5€. If the probability of winning is $2/3$, it is more likely that the gambler get ruined or that he/she reaches the target?

What happens if the probability of winning drops to $1/2$?

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = y^2 \\ \dot{y} = \alpha x + y - z^2 \\ \dot{z} = x - z^2 \end{cases}$$

for $\alpha \in \mathbb{R}$.

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{2}{5}Y + 2G - 2 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - \frac{1}{2}x_{h+1} - x_{h+1}x_h - \frac{1}{4}x_h = 0.$$