

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **January 19th, 2024**

1. The dynamics of a price in a Goodwin model is described by the equation

$$2p_{h+2} + 2\left(\alpha + \frac{1}{3}\right)p_{h+1} - 2\alpha p_h = 8, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case $\alpha = 1/3$ discuss the stability and write the explicit solutions.

2. A gambler starts from 3€ and has a target of winning 6€. If the probability of winning is $3/4$, it is more likely that the gambler get ruined or that he/she reaches the target?

What happens if the probability of winning drops to $1/2$?

3. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = \alpha x - y \\ \dot{y} = 2\alpha x - 2\alpha y \end{cases}$$

for $\alpha \in \mathbb{R}$. Are there any saddle, node or focus?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{2}{5}Y + 2G - 2 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+1} = \frac{1 + \alpha x_h^2}{x_h}$$

for $\alpha \in \mathbb{R}$.