

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **February 9th, 2024**

1. The dynamics of a price in a Goodwin model is described by the equation

$$3p_{h+2} + (\alpha + 1)p_{h+1} - \alpha p_h = 2, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case $\alpha = 1$ discuss the stability and write the explicit solutions.

2. A gambler starts from 3€ and has a target of winning 9€. If the probability of winning is $3/4$, it is more likely that the gambler get ruined or that he/she reaches the target?

What happens if the probability of winning drops to $1/2$?

3. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = x + y \\ \dot{y} = \alpha x + y \end{cases}$$

for $\alpha \in \mathbb{R}$. Are there any saddle, node or focus?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{1}{3}Y + 3G - 1 \\ \dot{G} = -kG - k \int_0^t Y(\tau) d\tau \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+1} = \alpha x_h (1 - \alpha x_h^2)$$

for $\alpha \in \mathbb{R}$.