

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **September 27th, 2024**

1. The dynamics of a price in a Goodwin model is described by the equation

$$2p_{h+2} + \alpha p_{h+1} - \alpha p_h = 2, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case $\alpha = 1$ discuss the stability and write the explicit solutions.

2. A gambler starts from 4€ and has a target of winning 8€. If the probability of winning is 1/4, what is the probability for the gambler to get ruined?

What happens if the probability of winning raises to 3/4?

3. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = x - y \\ \dot{y} = 2x + \alpha y \end{cases}$$

for $\alpha \in \mathbb{R}$. Are there any saddle, node or focus?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -Y + 3G - 2 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - \frac{3}{\alpha} x_{h+1}^2 + 2x_h = 0$$

for $\alpha \neq 0$.