

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance December 9th, 2024

1. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - 2x_{h+1} - x_{h+1}x_h + 2x_h = 0.$$

2. A gambler starts from 3€ and has a target of winning 5€. If the probability of winning is $1/3$, what is the probability of getting ruined?

What happens if the probability of winning raises to $1/2$?

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = -x + y \\ \dot{y} = -x^3 + \alpha y \end{cases}$$

for $\alpha \neq 0$. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -2Y + 3G - 3 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau - \frac{1}{3}Y \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

5. The dynamics of a price in a Goodwin model is described by the equation

$$\alpha p_{h+2} + (3 - \alpha)p_{h+1} - p_h = 6, \quad 0 < \alpha < 3.$$

Find the equilibrium position and discuss its stability.

In the case $\alpha = 2$ discuss the stability writing the explicit solutions.