

DSF 10/1/25

$$1) \quad x_{h+2} - 2x_{h+1} - 3x_h + 4 = 0$$

$$-4\bar{x} + 4 = 0 \quad \bar{x} = 1$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\lambda = -1$$

$$\lambda = 3$$

$$\bar{x} = 1 \text{ UNST.}$$

$$x_h = c_1 3^h + c_2 (-1)^h + 1$$

$$x_0 = 0$$

$$c_1 + c_2 + 1 = 0$$

$$x_1 = -2$$

$$3c_1 - c_2 + 1 = -2$$

$$4c_1 + 2 = -2$$

$$4c_1 = -4 \quad c_1 = -1$$

$$c_2 = -1 - c_1 = 0$$

$$x_h = -3^h + 1$$

□

$$2) \quad q = \frac{2}{3} \quad k=2 \quad N=6$$

$$x = \frac{kq}{q} = \frac{1}{2}$$

$$p_k = \frac{x^k - x^N}{1 - x^N}$$

$$p_2 = \frac{\frac{1}{4} - \frac{1}{64}}{1 - \frac{1}{64}} = \frac{16 - 1}{63} = \frac{15}{63} = \frac{5}{21} = 23,8\%$$

$$? N: \quad p_2 = 50\% \quad \text{for } q = \frac{1}{2}: \quad 1 - \frac{2}{N} = \frac{1}{2}$$
$$\Rightarrow N = 4 \quad \square$$

$$3) \quad \begin{aligned} -x^3 + \alpha y &= 0 \\ -x + \alpha y &= 0 \end{aligned} \quad \begin{aligned} -x^3 + x &= 0 \\ \alpha y &= x \end{aligned} \quad \begin{aligned} x(x^2-1) &= 0 \quad \begin{cases} x=0 \\ x=\pm 1 \end{cases} \\ y &= \begin{cases} 0 \\ \pm \frac{1}{\alpha} \end{cases} \end{aligned}$$

$$\begin{bmatrix} -3x^2 & \alpha \\ -1 & \alpha \end{bmatrix} \quad (0,0) \quad \begin{bmatrix} 0 & \alpha \\ -1 & \alpha \end{bmatrix} \quad \begin{aligned} \tau &= \alpha & \alpha < 0 \text{ SADDLE} \\ \delta &= \alpha & \alpha > 0 \end{aligned}$$

$$(0,0) \quad \alpha < 0 \text{ SADDLE}$$

$$0 < \alpha < 4 \quad \text{UNST. FOCUS}$$

$$\alpha \geq 4 \quad \text{UNST. NODE}$$

$$\tau^2 - 4\delta = \alpha^2 - 4\alpha \geq 0 \quad \alpha(\alpha-4) \geq 0 \quad \alpha \leq 0 \vee \alpha \geq 4$$

$$(\pm 1, \pm \frac{1}{\alpha}) \quad \begin{bmatrix} -3 & \alpha \\ -1 & \alpha \end{bmatrix} \quad \begin{aligned} \tau &= \alpha - 3 \\ \delta &= -2\alpha \end{aligned}$$

$$\tau^2 - 4\delta = (\alpha-3)^2 + 8\alpha \geq 0$$

$$\alpha^2 - 6\alpha + 9 + 8\alpha = \alpha^2 + 2\alpha + 9 \geq 0 \quad \forall \alpha$$

$$\alpha < 0 \quad \text{ST. NODES}$$

$$\alpha > 0 \quad \text{SADDLES}$$



$$4) \quad \begin{cases} \dot{y} = -2y + 3z - 3 \\ \dot{z} = z \\ \dot{x} = -2x - ky \end{cases}$$

$$\bar{y} = 1$$

$$\bar{z} = 0 \quad (0, 1, 0)$$

$$\bar{x} = 0$$

$$k > 0$$

$$\det \begin{bmatrix} -2-\lambda & 3 & 0 \\ 0 & -\lambda & 1 \\ -k & 0 & -2-\lambda \end{bmatrix} = \lambda(\lambda+2)(-\lambda-2) - 3k$$

$$= -\lambda(\lambda^2 + 4\lambda + 4) - 3k$$

$$\lambda^3 + 4\lambda^2 + 4\lambda + 3k$$

$$\text{Necessary cond. ok}$$

$$\left[\begin{array}{cc|c} 4 & 3k & 0 \\ 1 & 4 & 0 \\ 0 & 4 & 3 \end{array} \right] \quad 16 - 3k > 0 \quad k < \frac{16}{3} \quad \text{G.A.S.}$$



$$5) \quad y_{h+1} - \frac{1+\beta}{3} y_h + \frac{\beta}{3} y_{h-1} = 1 \quad \beta > 0$$

$$\bar{y} - \frac{1+\beta}{3} \bar{y} + \frac{\beta}{3} \bar{y} = 1 \quad \frac{2}{3} \bar{y} = 1 \quad \bar{y} = \frac{3}{2}$$

$$\lambda^2 - \frac{1+\beta}{3} \lambda + \frac{\beta}{3}$$

$$\frac{1+\beta}{3} < 1 + \frac{\beta}{3} < 2$$

$$\Rightarrow \frac{\beta}{3} < 1 \quad \boxed{\beta < 3} \text{ G.A.S.}$$

$$\begin{aligned} \Delta &= \left(\frac{1+\beta}{3} \right)^2 - \frac{4\beta}{9} = \frac{1+\beta^2+2\beta}{9} - \frac{4\beta}{9} = \frac{1+\beta^2+2\beta-4\beta}{9} \\ &= \frac{1+\beta^2-2\beta}{9} < 0 \end{aligned}$$

$$\beta_{1/2} = 5 \pm \sqrt{25-1} = 5 \pm 2\sqrt{6}$$

oscillations for $5-2\sqrt{6} < \beta < 5+2\sqrt{6}$

For $5-2\sqrt{6} < \beta < 3$ STABLE OSCILL.

$3 < \beta < 5+2\sqrt{6}$ UNSTABLE OSCILL.