

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance

Jan 10th, 2025

1. Discuss the stability of the equilibria of the linear non-homogeneous discrete equation

$$x_{h+2} - 2x_{h+1} - 3x_h + 4 = 0$$

and find the explicit solution such that $x_0 = 0$ and $x_1 = -2$.

2. A gambler starts from 2€ and has a target of winning 6€. If the probability of winning is $2/3$, what is the probability of getting ruined?

If the probability of winning drops to $1/2$, what is the value N of the target for which the probability of ruining is exactly 50%?

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = -x^3 + \alpha y \\ \dot{y} = -x + \alpha y \end{cases}$$

for $\alpha \neq 0$. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -2Y + 3G - 3 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

5. The equation of a Samuelson's accelerator model is given by

$$Y_{h+1} - \frac{1}{3}(1 + \beta)Y_h + \frac{\beta}{3}Y_{h-1} = 1, \quad \beta > 0.$$

Find the equilibrium position and discuss its stability. Moreover, discuss the existence of oscillations.