

DSF 7/2/25

$$d \neq 0$$

$$2) \begin{cases} \dot{x} = x^3 y^2 + x \\ \dot{y} = -x + d(y - y^2) \end{cases} \quad \begin{aligned} x(x^2 y^2 + 1) &= 0 \quad x=0 \\ d(y - y^2) &= 0 \quad \begin{cases} y=0 \\ y=1 \end{cases} \end{aligned}$$

$$\begin{bmatrix} 3x^2 y^2 + 1 & 2x^3 y \\ -1 & d(1 - 2y) \end{bmatrix}$$

$$(0,0) \quad \begin{bmatrix} 1 & 0 \\ -1 & d \end{bmatrix}$$

$$\text{UNST} \quad \begin{cases} d > 0 \quad \text{NODE} \\ d < 0 \quad \text{SADDLE} \end{cases}$$

$$(0,1) \quad \begin{bmatrix} 1 & 0 \\ -1 & -d \end{bmatrix}$$

$$\text{UNST} \quad \begin{cases} d < 0 \quad \text{NODE} \\ d > 0 \quad \text{SADDLE} \end{cases}$$

$$3) \quad \ddot{G} = -2\dot{G} - \gamma$$

$$\begin{cases} \dot{\gamma} = -k\gamma + G - 1 \\ \dot{G} = Z \\ \dot{Z} = -\gamma - 2Z \end{cases} \quad \begin{bmatrix} -k-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ -1 & 0 & -2-\lambda \end{bmatrix}$$

$$\lambda(\lambda+2)(-k-\lambda) - 1 \Rightarrow (\lambda^2 + 2\lambda)(-k-\lambda) + 1$$

$$\lambda^3 + (k+2)\lambda^2 + 2k\lambda + 1$$

$$\left[\begin{array}{cc|c} k+2 & 1 & 0 \\ 1 & 2k & 0 \\ \hline 0 & k+2 & 1 \end{array} \right] \quad \begin{array}{l} 2k(k+2) > 1 \\ 2k^2 + 4k - 1 > 0 \end{array}$$

$$k = \frac{-2 \pm \sqrt{4+2}}{2} = -1 \pm \frac{\sqrt{6}}{2}$$

$$k > \frac{\sqrt{6}}{2} - 1 \quad \text{ST}$$

$$0 < k < \frac{\sqrt{6}}{2} - 1 \quad \text{UNST.} \quad \square$$

$$4) \quad y_{n+1} - \frac{7}{2} y_n + 3 y_{n-1} = 1$$

$$\left(1 - \frac{7}{2} + 3\right) \bar{y} = 1 \quad \frac{2-7+6}{2} \bar{y} = 1 \quad \bar{y} = 2$$

$$\lambda^2 - \frac{7}{2} \lambda + 3 = 0 \quad 2\lambda^2 - 7\lambda + 6 = 0$$

$$\frac{7 \pm \sqrt{49-48}}{4} = \begin{array}{l} \frac{3}{2} \\ 2 \end{array} \quad \text{UNST}$$

$$y_n = c_1 \left(\frac{3}{2}\right)^n + c_2 2^n + 2$$

$$\begin{cases} y_0 = c_1 + c_2 + 2 = 0 & -3c_2 = -2 \quad c_2 = \frac{2}{3} \\ y_1 = \frac{3}{2} c_1 + 2c_2 + 2 = 2 & c_1 = -4c_2 \quad c_1 = -\frac{8}{3} \end{cases}$$

$$y_n = -\frac{8}{3} \left(\frac{3}{2}\right)^n + \frac{2}{3} \cdot 2^n + 2 \quad \square$$

$$5) \quad x = \frac{1-9}{9} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$p_1 = \frac{\left(\frac{1}{2}\right)^1 - \left(\frac{1}{2}\right)^4}{1 - \left(\frac{1}{2}\right)^4} = \frac{\frac{1}{2} \left(1 - \frac{1}{8}\right)}{1 - \frac{1}{16}} = \frac{7}{16} \cdot \frac{16}{15} = \frac{7}{15} = 46,67\%$$

$$\left[\begin{array}{l} 1 - \frac{1}{N} = \frac{a}{b} \quad \frac{1}{N} = 1 - \frac{a}{b} = \frac{b-a}{b} \\ N = \frac{b}{b-a} \quad b=10 \\ \quad \quad \quad b-a=2 \quad a=8 \end{array} \right]$$

$$1 - \frac{1}{N} = 80\% = \frac{4}{5} \quad \frac{1}{N} = \frac{1}{5} \quad N=5$$

□

$$1) \quad \begin{cases} x_{h+1} = x_h(2 - ky_h) & x(2 - ky) = x \\ y_{h+1} = y_h(x_h - 1) & y(x - 1) = y \end{cases}$$

$$\begin{array}{ll} x=0 & x=1 \\ y=0 & y=\frac{2}{k} \quad k \neq 0 \end{array}$$

$$\begin{bmatrix} 2-ky & -kx \\ y & x-1 \end{bmatrix} \quad (0,0) \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{UNST.}$$

$$\left(1, \frac{2}{k}\right) \quad \begin{bmatrix} 0 & -2 & -k \\ \frac{2}{k} & 0 & 1 \end{bmatrix} \quad \lambda^2 + 2 = 0 \quad \lambda = \pm i\sqrt{2} \quad \text{UNST.} \quad \square$$