

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **Feb 7th, 2025**

1. A nonlinear DDS is of the form

$$\begin{cases} x_{h+1} = x_h(2 - ky_h) \\ y_{h+1} = y_h(x_h - 1) \end{cases}$$

for $k \in \mathbb{R}$. Find the equilibria and discuss their stability.

2. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = x^3y^2 + x \\ \dot{y} = -x + \alpha(y - y^2) \end{cases}$$

for $\alpha \neq 0$. Are there any saddles, nodes or foci?

3. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -kY + G - 1 \\ \dot{G} = -2G - \int_0^t Y(\tau)d\tau \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

4. The equation of a Samuelson's accelerator model is given by

$$Y_{h+1} - \frac{7}{2}Y_h + 3Y_{h-1} = 1.$$

Find the equilibrium position and discuss its stability. Moreover, find the explicit solution such that $Y_0 = 0$ and $Y_1 = 2$.

5. A gambler starts from 1€ and has a target of winning 4€. If the probability of winning is $2/3$, what is the probability of getting ruined?

If the probability of winning drops to $1/2$, what is the value N of the target for which the probability of ruining is exactly 80%?