

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **July 18th, 2025**

1. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - 2x_{h+1} - x_{h+1}x_h + 2x_h = 0.$$

2. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = \alpha(x - y) \\ \dot{y} = 2x - y \end{cases}$$

for $\alpha \in \mathbb{R}$, finding all the saddles, nodes and foci.

3. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -kY + G - 2 \\ \dot{G} = -3G - \int_0^t Y(\tau) d\tau \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

4. Discuss the stability of the equilibria of the d.d.s.

$$\begin{cases} x_{h+1} = -x_h^2 - \alpha y_h \\ y_{h+1} = -x_h + y_h \end{cases}$$

for $\alpha \in \mathbb{R}$.

5. A gambler starts from 2€ and has a target of winning 5€. If the probability of winning is $2/3$, what is the probability of getting ruined?

If the probability of winning drops to $1/2$, what is the value N of the target for which the probability of ruining is exactly 50%?