

Dec 3, 2025

$$1) \quad q = \frac{2}{5} \quad x = \frac{\frac{3}{5}}{\frac{2}{5}} = \frac{3}{2}$$

$$P_h = \frac{x^h - x^N}{1 - x^N} = \frac{\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^5}{1 - \left(\frac{3}{2}\right)^5} = \frac{2^2 \cdot 3^2 - 3^5}{2^5 - 3^5}$$

$$= \frac{3^5 - 2^2 \cdot 3^2}{3^5 - 2^5} = \frac{243 - 72}{243 - 32} = \frac{171}{211} \sim 81\%$$

$$q = \frac{1}{2} \quad x = 1$$

$$P_h = \frac{1}{2} \Rightarrow 1 - \frac{2}{N} = \frac{1}{2} \Rightarrow N = 4$$

$$P_a = 1 - \frac{h}{N} = 1 - \frac{2}{5}$$

$$= \frac{3}{5} = 60\%$$

$$2) \quad x_{h+2} - x_{h+1}^2 - \alpha x_{h+1} x_h + \frac{1}{2} x_h = 0 \quad \alpha \geq 0$$

$$x - x^2 - \alpha x^2 + \frac{1}{2} x = 0$$

$$x = 0$$

$$\frac{3}{2} - (1 + \alpha)x = 0$$

$$x = \frac{3}{2 + 2\alpha}$$

$$\begin{cases} x_{h+1} = y_h \\ y_{h+1} = -\frac{1}{2} x_h + y_h^2 + \alpha x_h y_h \end{cases} \quad \begin{matrix} (0, 0) \\ \left(\frac{3}{2+2\alpha}, \frac{3}{2+2\alpha}\right) \end{matrix}$$

$$\begin{bmatrix} 0 & 1 \\ -\frac{1}{2} + \alpha y & 2y + \alpha x \end{bmatrix}$$

$$(0, 0)$$

$$\begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 0 \end{bmatrix}$$

$$\lambda^2 + \frac{1}{2} = 0$$

$$\rho(A) = \frac{1}{\sqrt{2}} < 1 \Rightarrow 0 \text{ A.S.}$$

$$\left(\frac{3}{2+2\alpha}, \frac{3}{2+2\alpha} \right)$$

$$\begin{bmatrix} 0 & 1 \\ -\frac{1}{2} + \frac{3\alpha}{2+2\alpha} & \frac{(2+\alpha)3}{2+2\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2\alpha-1}{2+2\alpha} & \frac{3(2+\alpha)}{2+2\alpha} \end{bmatrix}$$

$$\rho = \frac{3(2+\alpha)}{2+2\alpha} > 0$$

$$\delta = \frac{1-2\alpha}{2+2\alpha}$$

$$|\alpha| < 1 + \delta < 2$$

$$\begin{cases} \frac{3(2+\alpha)}{2+2\alpha} < 1 + \frac{1-2\alpha}{2+2\alpha} \Rightarrow 6+3\alpha < 3 \Rightarrow \alpha < -2 \text{ NO} \\ \frac{1-2\alpha}{2+2\alpha} < 1 \end{cases}$$

$$\frac{3}{2+2\alpha} \text{ UNST.}$$

$$3) \begin{cases} \dot{D} = D(1-D-kS) \\ \dot{S} = S(-k+D) \end{cases} \quad k \neq 0$$

$$S=0 \quad D=0, D=1$$

$$D=k \quad 1-k-kS=0 \quad S = \frac{1-k}{k}$$

$$(0, 0) \quad (1, 0) \quad \left(k, \frac{1-k}{k}\right)$$

$$\begin{bmatrix} 1-2D-kS & -kD \\ S & D-k \end{bmatrix}$$

$$(0,0) \quad \begin{bmatrix} 1 & 0 \\ 0 & -k \end{bmatrix} \quad \begin{array}{l} k > 0 \text{ SADDLE} \\ k < 0 \text{ UNST. NODE} \end{array}$$

$$(1,0) \quad \begin{bmatrix} -1 & -k \\ 0 & 1-k \end{bmatrix} \quad \begin{array}{l} k < 1 \text{ SADDLE} \\ k > 1 \text{ ST. NODE} \end{array}$$

$$\left(k, \frac{1-k}{k}\right) \quad \begin{bmatrix} -k & -k^2 \\ \frac{1-k}{k} & 0 \end{bmatrix} \quad \begin{array}{l} r = -k \\ s = k(1-k) \end{array}$$

$$k < 0 \vee k > 1 \text{ SADDLE}$$

$$0 < k < \frac{4}{5} \text{ ST. FOCUS}$$

$$\frac{4}{5} \leq k < 1 \text{ ST. NODE}$$

$$k^2 - 4k + 4k^2 \geq 0$$

$$5k^2 - 4k \geq 0 \quad k \geq \frac{4}{5}$$

$$4) \quad \begin{cases} \dot{Y} = -kY + 3G - 3 \\ \dot{G} = -G - k \int_0^t Y(\tau) d\tau - Y \end{cases} \quad k > 0$$

$$\begin{cases} \dot{Y} = -kY + 3G - 3 \\ \dot{G} = Z \\ \dot{Z} = -Z - \cancel{kY} + \cancel{kY} - 3G + 3 \end{cases}$$

$$\begin{cases} -k\gamma + 3G - 3 = 0 & \Rightarrow \gamma = 0 \\ z = 0 \\ -z - 3G + 3 = 0 & \Rightarrow G = 1 \end{cases}$$

$$(0, 1, 0)$$

$$\det \begin{bmatrix} -k-\lambda & 3 & 0 \\ 0 & 0-\lambda & 1 \\ 0 & -3 & -1-\lambda \end{bmatrix} = (1+\lambda)(k+\lambda)(-\lambda) - 3(k+\lambda)$$

$$(k+\lambda)(-\lambda(1+\lambda) - 3) = 0$$

$$\lambda = -k \quad \underbrace{-\lambda^2 - \lambda - 3}_p \quad \text{A.S.T.}$$

$$5) \quad x_{h+1} = x_h^3 - 2x_h^2 + x_h \quad \alpha \in \mathbb{R}$$

$$x^3 - 2x^2 + x = x$$

$$x^2(x-2) = 0 \quad \begin{cases} x=0 \\ x=2 \end{cases} \quad \alpha \neq 0$$

$$f'(x) = 3x^2 - 2\alpha x + 1 \quad f''(x) = 6x - 2\alpha$$

$$f'(0) = 1 \quad f''(0) = -2\alpha \quad \begin{cases} \alpha \neq 0 & \text{UNST} \\ \alpha = 0 & f'''(0) > 0 \\ & \text{UNST} \end{cases}$$

$$f'''(x) = 6$$

$$\alpha \neq 0$$

$$f'(\alpha) = 3\alpha^2 - 2\alpha^2 + 1 = \alpha^2 + 1 > 1 \quad \text{UNST.}$$

