

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance December 3, 2025

1. A gambler starts from 2€ and aims to reach a total of 5€. If the probability of winning is $\frac{2}{5}$, what is the probability that the gambler gets ruined?

What happens if the probability of winning raises to $1/2$? What would be the target N needed to have exactly a 50% chance of getting ruined?

2. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - x_{h+1}^2 - \alpha x_{h+1}x_h + \frac{1}{2}x_h = 0$$

for $\alpha > 0$.

3. The variations of supply and demand of a good follow the system of ODEs

$$\begin{cases} \dot{D} = D(1 - D - kS) \\ \dot{S} = S(D - k) \end{cases}$$

for $k \neq 0$.

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -kY + 3G - 3 \\ \dot{G} = -G - k \int_0^t Y(\tau) d\tau - Y \end{cases}$$

for $k > 0$.

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h^3 - \alpha x_h^2 + x_h$$

for $\alpha \in \mathbb{R}$.