

DSF

16/1/26

$$1) \quad q = \frac{2}{3} \quad N = 7 \quad h = 4$$

$$P_4 = \frac{x^4 - x^7}{1 - x^7}$$

$$x = \frac{1-q}{q} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$= \frac{\frac{1}{2^4} - \frac{1}{2^7}}{1 - \frac{1}{2^7}} = \frac{2^3 - 1}{2^7 - 1} = \frac{7}{127} = 0,055$$

$$= 5,5\%$$

$$q = \frac{1}{2} \quad P_4 = 1 - \frac{4}{7} = \frac{3}{7} = 42,86\%$$

$$1 - \frac{4}{N} = \frac{1}{2} \quad N = 8$$

$$2) \quad \cancel{x} - \cancel{x} - \alpha x + 1 = 0 \quad x = \frac{1}{\alpha} \quad \alpha \neq 0$$

$\alpha = 0$ NO EQ. SOL.

$$\alpha \neq 0 \quad \alpha^2 - \alpha - \alpha = 0 \quad 1 < 1 - \alpha < 2$$

$$0 < -\alpha < 1 \quad -1 < \alpha < 0$$

$$F_{01} \quad -1 < \alpha < 0 \quad \frac{1}{\alpha} \text{ G.A.S.}$$

$$F_{01} \quad \alpha < -1 \vee \alpha > 0 \quad \frac{1}{\alpha} \text{ UNST}$$

$$\lambda = -1$$

$$\lambda^2 - \lambda + 1 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$|\lambda| = 1$$

$$\lambda_1 \neq \lambda_2 \Rightarrow \text{SIMPLY STABLE}$$

$$\lambda = 2$$

$$\lambda^2 - \lambda - 2 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1+8}}{2} = \frac{-1}{2}$$

$$x_h = c_1 (-1)^h + c_2 2^h + \frac{1}{2}$$

$$x_0: \begin{cases} c_1 + c_2 + \frac{1}{2} = 1 \\ -c_1 + 2c_2 + \frac{1}{2} = 3 \end{cases} \quad c_1 = \frac{1}{2} - c_2$$

$$-\frac{1}{2} + c_2 + 2c_2 + \frac{1}{2} = 3 \quad c_2 = 1$$

$$c_1 = -\frac{1}{2}$$

$$\boxed{x_h = -\frac{1}{2} (-1)^h + 2^h + \frac{1}{2}}$$

$$x_0 = -\frac{1}{2} + 1 + \frac{1}{2} = 1 \quad \checkmark$$

$$x_1 = \frac{1}{2} + 2 + \frac{1}{2} = 3$$

□

$$3) \begin{cases} \dot{D} = kD(1+S) \\ \dot{S} = -kD - S \end{cases}$$

$$\begin{cases} kD(1+S) = 0 \\ -kD - S = 0 \end{cases}$$

$$k \neq 0 \Rightarrow \begin{cases} D=0 \Rightarrow S=0 \\ S=-1 \Rightarrow -kD+1=0 \end{cases}$$

$$(0,0) \quad \left(\frac{1}{k}, -1\right) \quad D = \frac{1}{k}$$

$$\nabla F = \begin{bmatrix} k(1+S) & kD \\ -k & -1 \end{bmatrix}$$

$(0,0)$

$$\nabla F(0,0) = \begin{bmatrix} k & 0 \\ -k & -1 \end{bmatrix} \quad \begin{array}{l} k > 0 \text{ UNS. SADDLE} \\ k < 0 \text{ A.S. NODE} \end{array}$$

$$\nabla F\left(\frac{1}{k}, -1\right) = \begin{bmatrix} 0 & 1 \\ -k & -1 \end{bmatrix} \quad \begin{array}{l} \rho = -1 \\ \delta = k \end{array}$$

$$\underbrace{\lambda^2 + \lambda + k}_p$$

$$\begin{array}{ll} k < 0 & \text{v. UNST.} \\ k > 0 & \text{p. A.S.} \end{array}$$

$$\left(\frac{1}{k}, -1\right) \quad D = 1 - 4k \geq 0 \quad k \leq \frac{1}{4}$$

$$0 < k \leq \frac{1}{4} \quad \begin{array}{l} \text{ST.} \\ \text{NODE} \end{array}$$

$$k > \frac{1}{4} \quad \begin{array}{l} \text{ST.} \\ \text{FOCUS} \end{array}$$

$$k < 0 \quad \text{SADDLE}$$



$$4) \quad \ddot{G} = -\dot{G} - kY - k(-Y + 3G - 3) \\ = -\dot{G} - \cancel{kY} + \cancel{kY} - 3kG + 3k$$

$$\begin{cases} \dot{Y} = -Y + 3G - 3 \\ \dot{G} = Z \\ \dot{Z} = -3kG - Z + 3k \end{cases} \quad (0, 1, 0)$$

$$\det \begin{bmatrix} -1-\lambda & 3 & 0 \\ 0 & 0-\lambda & 1 \\ 0 & -3k & -1-\lambda \end{bmatrix} = (1+\lambda)^2(-\lambda) - 3k(1+\lambda) \\ = (1+\lambda) \left[-\lambda^2 - \lambda - 3k \right]$$

$$\lambda = -1$$

$$\underbrace{\lambda^2 + \lambda}_{p} + \underbrace{3k}_{p} = 0$$

A.S.



$$5) \quad x_{h+1} = \frac{x_h}{x_h^2 + \alpha} \quad \alpha > 0, \alpha \neq 1$$

$$\frac{x}{x^2 + \alpha} = x$$

$$x = 0$$

$$\frac{1}{x^2 + \alpha} = 1$$

$$x^2 + \alpha = 1$$

$$x^2 = 1 - \alpha$$

$$x = \pm \sqrt{1 - \alpha} \quad 0 \leq \alpha < 1$$

$$f'(x) = \frac{x^2 + \alpha - 2x^2}{(x^2 + \alpha)^2} = \frac{-x^2 + \alpha}{(x^2 + \alpha)^2}$$

$$f'(0) = \frac{1}{\alpha}$$

$$\alpha > 1 \quad \text{A.S.}$$

$$0 < \alpha < 1 \quad \text{UNST}$$

$$f'(\pm\sqrt{1-\alpha}) = \frac{-1 + \alpha + \alpha}{(1 - \alpha + \alpha)^2} = 2\alpha - 1$$

$$-1 < 2\alpha - 1 < 1$$

$$0 < 2\alpha < 2$$

$$0 < \alpha < 1 \quad \text{A.S.}$$

$$\alpha > 1 \quad \text{UNST.}$$

