

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **January 16th, 2026**

1. A gambler starts from 4€ and aims to reach a total of 7€. If the probability of winning is $\frac{2}{3}$, what is the probability that the gambler gets ruined?

What happens if the probability of winning lowers to $1/2$? In this case, what would be the target N needed to have exactly a 50% chance of getting ruined?

2. Discuss the stability of the linear second-order discrete equation

$$x_{h+2} - x_{h+1} - \alpha x_h + 1 = 0$$

for $\alpha \in \mathbb{R}$.

In the case $\alpha = 2$, find the explicit solution for $x_0 = 1$, $x_1 = 3$.

3. The variations of supply and demand of a good follow the system of ODEs

$$\begin{cases} \dot{D} = kD + kDS \\ \dot{S} = -kD - S \end{cases}$$

for $k \neq 0$.

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -Y + 3G - 3 \\ \dot{G} = -G - k \int_0^t Y(\tau) d\tau - kY \end{cases}$$

for $k > 0$.

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = \frac{x_h}{x_h^2 + \alpha}, \quad \alpha > 0, \alpha \neq 1.$$