

Feb 13, 2026

$$1) \quad q = 0.4 \quad x = \frac{1-q}{q} = \frac{0.6}{0.4} = \frac{3}{2}$$

$$p_{20} = \frac{\left(\frac{3}{2}\right)^{20} - \left(\frac{3}{2}\right)^{10}}{\left(\frac{3}{2}\right)^{20} - 1} = 0,983 = 98,3\%$$

$$q = 0.5 \quad p_{20} = 1 - \frac{h}{N} = 1 - \frac{10}{20} = \frac{1}{2} = 50\%$$

$$1 - \frac{10}{N} = \frac{60}{100} = \frac{3}{5} \quad \frac{10}{N} = \frac{2}{5} \quad N = \frac{5}{2} \cdot 10 = 25 \quad \square$$

$$2) \quad \lambda^2 - \frac{1}{2}\lambda + \alpha = 0 \quad \frac{1}{2} < 1 + \alpha < 2$$

$$-\frac{1}{2} < \alpha < 1 \quad \text{A.S.}$$

$$\bar{x} - \frac{1}{2}\bar{x} + \alpha\bar{x} - 1 = 0 \quad \left(\frac{1}{2} + \alpha\right)\bar{x} = 1$$

$$\bar{x} = \frac{1}{\alpha + \frac{1}{2}} = \frac{2}{2\alpha + 1} \quad \alpha \neq -\frac{1}{2}$$

$$\alpha = 0 \quad \lambda^2 - \frac{1}{2}\lambda = 0$$

$$\lambda = 0 \quad \lambda = \frac{1}{2}$$

$$x_h = c_2 \frac{1}{2^h} + 2$$

$$h \geq 1$$

$$h=1 \quad \frac{c_2}{2} + 2 = 5 \Rightarrow c_2 = 6$$

$$\begin{cases} x_h = \frac{6}{2^h} + 2 & h \geq 1 \\ x_0 = 2 \end{cases} \quad \square$$

$$3) \quad \begin{aligned} D(2-D-S) &= 0 \\ S(3-D-2S) &= 0 \end{aligned}$$

$$D=0 \Rightarrow \begin{cases} -S=0 \\ S=\frac{3}{2} \end{cases}$$

$$D+S=2 \quad \begin{cases} S=0 & D=2 \\ D+2S=3 & 2+S=3 & S=1 & D=1 \end{cases}$$

$$(0,0) \quad (0, \frac{3}{2}) \quad (2,0) \quad (1,1)$$

$$J = \begin{bmatrix} 2-2D-S & -D \\ -S & 3-D-4S \end{bmatrix}$$

$$J(0,0) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{UNST. NODE}$$

$$J(0, \frac{3}{2}) = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & -3 \end{bmatrix} \quad \text{UNST. SADDLE}$$

$$J(2,0) = \begin{bmatrix} -2 & -2 \\ 0 & 1 \end{bmatrix} \quad \text{UNST. SADDLE}$$

$$J(1,1) = \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}$$

ST-NODE

$$\tau = -3 \quad \delta = 1 \quad \tau^2 \geq 4\delta$$

$$\tau < 0$$

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$$4) \quad \dot{G} = Z \quad \ddot{G} = \dot{Z} = -Z - kY - \dot{Y}$$

$$\begin{cases} \dot{Y} = -Y + 2G - 6 \\ \dot{G} = Z \\ \dot{Z} = -kY + Y - 2G + 6 - Z = (1-k)Y - 2G - Z + 6 \end{cases}$$

$$\begin{aligned} \bar{Z} &= 0 & \bar{Y} &= 2\bar{G} - 6 & \bar{Y} &= (1-k)\bar{Y} \\ & & (1-k)\bar{Y} &= 2\bar{G} - 6 & k\bar{Y} &= 0 \quad \bar{Y} = 0 \\ \bar{G} &= 3 & & & & \end{aligned}$$

$(0, 3, 0)$

$$\begin{bmatrix} -1-\lambda & 2 & 0 \\ 0 & 0-\lambda & 1 \\ 1-k & -2 & -1-\lambda \end{bmatrix} - \lambda(1+\lambda)^2 + 2(1-k) - 2(\lambda+1) =$$

$$= -\lambda - \lambda^3 - 2\lambda^2 - 2\lambda - 2 + 2 - 2k$$

$$= -\lambda^3 - 2\lambda^2 - 3\lambda - 2k$$

$$\lambda^3 + 2\lambda^2 + 3\lambda + 2k$$

$$\left[\begin{array}{cc|c} 2 & 2k & 0 \\ 1 & 3 & 0 \\ \hline 0 & 2 & 2k \end{array} \right]$$

$$6 - 2k > 0$$

$$2k < 6 \quad \boxed{k < 3}$$

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5) $x_{h+1} = \frac{\alpha x_h}{1+x_h^2} \quad \alpha > 0 \quad \frac{\alpha x}{1+x^2} = x \quad \begin{array}{l} x=0 \\ x^2+1=\alpha \\ x^2=\alpha-1 \end{array}$

$$f'(x) = \frac{\alpha(1+x^2) - 2x^2\alpha}{(1+x^2)^2}$$

$$x = \pm \sqrt{\alpha-1} \quad \alpha > 1$$

$$= \frac{\alpha - \alpha x^2}{(1+x^2)^2} = \frac{\alpha(1-x^2)}{(1+x^2)^2}$$

$$f'(0) = \alpha \quad 0 < \alpha < 1 \quad \text{A.S.}$$

$$\alpha > 1 \quad \text{UNST}$$

$$\alpha > 1$$

$$f'(\pm\sqrt{\alpha-1}) = \frac{\alpha(1-(\alpha-1))}{(1+\alpha-1)^2} = \frac{\cancel{\alpha}(1-\alpha+1)}{\alpha^2} = \frac{2-\alpha}{\alpha}$$

$$-1 < \frac{\alpha-2}{\alpha} < 1$$

$$-\alpha < \alpha-2 < \alpha$$

$$\begin{cases} 2\alpha > 2 \Rightarrow \alpha > 1 \\ -2 < 0 \quad \vee \end{cases}$$

$$\alpha > 1 \quad \text{A.S.}$$

$$0 < \alpha < 1 \quad \text{UNST.}$$

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