

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance

February 13, 2026

1. A gambler starts from 10€ and aims to reach a total of 20€. If the probability of winning is 0.4, what is the probability that the gambler gets ruined?

What happens if the probability of winning raises to 1/2? In this case, what would be the target N needed to have exactly a 60% chance of getting ruined?

2. Discuss the stability of the linear second-order discrete equation

$$x_{h+2} - \frac{1}{2}x_{h+1} + \alpha x_h - 1 = 0$$

for $\alpha \in \mathbb{R}$.

In the case $\alpha = 0$, find the explicit solution for $x_0 = 2$, $x_1 = 5$.

3. The variations of supply and demand of a good follow the system of ODEs

$$\begin{cases} \dot{D} = D(2 - D - S) \\ \dot{S} = S(3 - D - 2S). \end{cases}$$

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -Y + 2G - 6 \\ \dot{G} = -G - k \int_0^t Y(\tau) d\tau - Y \end{cases}$$

for $k > 0$.

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = \frac{\alpha x_h}{x_h^2 + 1}, \quad \alpha > 0.$$

We recall that the derivative of a rational function $\frac{f(x)}{g(x)}$ is given by

$$\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$