

DSF 8/6/26

$$1) \quad x = \frac{1-p}{p} = \frac{0.55}{0.45} = \frac{11}{9}$$

$$P = \frac{\left(\frac{11}{9}\right)^{15} - \left(\frac{11}{9}\right)^4}{\left(\frac{11}{9}\right)^{15} - 1} = 93,6\%$$

$$p = \frac{1}{2} \quad P = 1 - \frac{k}{N} = 1 - \frac{4}{15} = \frac{11}{15} = 73,3\%$$

$$1 - \frac{4}{N} = 0,6 \quad \frac{4}{N} = 0,4 \quad N = \frac{4}{0,4} = 10$$

$$2) \quad \bar{x} - \frac{3}{2} \bar{x} + \alpha \bar{x} = 3 \quad \left(\alpha - \frac{1}{2}\right) \bar{x} = 3$$

$$\bar{x} = \frac{6}{2\alpha - 1} \quad \alpha \neq \frac{1}{2} \quad \alpha = \frac{1}{2} \text{ no eq.}$$

$$\lambda^2 - \frac{3}{2} \lambda + \alpha$$

$$\frac{3}{2} < 1 + \alpha < 2$$

$$\frac{1}{2} < \alpha < 1 \quad \text{A.S.}$$

$$\alpha < \frac{1}{2} \vee \alpha > 1 \quad \text{UNST.}$$

$$\alpha = \frac{1}{2} \quad \text{no eq.}$$

$$\alpha = 1 \quad 2\lambda^2 - 3\lambda + 2 = 0$$

$$\frac{3 \pm \sqrt{9 - 16}}{4} \begin{cases} \frac{3}{4} + i \frac{\sqrt{7}}{4} \\ \frac{3}{4} - i \frac{\sqrt{7}}{4} \end{cases}$$

$$\left| \frac{3}{4} \pm i \frac{\sqrt{7}}{4} \right| = \sqrt{\frac{9}{16} + \frac{7}{16}} = 1$$

$$\bar{x} = 6 \quad \text{SIMPLY STABLE}$$

$$\alpha = -1 \quad \bar{x} = \frac{6}{-3} = -2.$$

$$2\lambda^2 - 3\lambda - 2 = 0 \quad \frac{3 \pm \sqrt{9+16}}{4} \begin{matrix} 2 \\ -\frac{1}{2} \end{matrix}$$

$$x_h = c_1 2^h + c_2 \left(-\frac{1}{2}\right)^h - 2$$

$$\begin{cases} c_1 + c_2 - 2 = 0 \end{cases}$$

$$c_1 = 2 - c_2$$

$$\begin{cases} 2c_1 - \frac{1}{2}c_2 - 2 = 0 \end{cases}$$

$$4 - 2c_2 - \frac{1}{2}c_2 - 2 = 0$$

$$\frac{5}{2}c_2 = 2 \quad c_2 = \frac{4}{5}$$

$$c_1 = \frac{10-4}{5} = \frac{6}{5}$$

$$x_h = \frac{6}{5} 2^h + \frac{4}{5} \left(-\frac{1}{2}\right)^h - 2$$

$$3) \quad P=0 \begin{cases} S=0 \\ S=3 \end{cases}$$

$$P+S=2 \begin{cases} S=0 & P=2 \\ 2P+S=3 & P=1, S=1 \end{cases}$$

$$(0,0) \quad (0,3) \quad (2,0) \quad (1,1)$$

$$\nabla F = \begin{bmatrix} 2-2P-S & -P \\ -2S & 3-2P-2S \end{bmatrix}$$

$$\nabla F(0,0) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

UNST.
NODE

$$\nabla F(0,3) = \begin{bmatrix} -1 & 0 \\ -6 & -3 \end{bmatrix}$$

ST. NODE

$$\nabla F(2,0) = \begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$$

ST. NODE

$$\nabla F(1,1) = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

$$\rho = -2$$

$$\delta = -1$$

SADDLE

$$4) \quad \dot{Y} = -2Y + G - 1$$

$$\dot{G} = Z$$

$$\ddot{G} = -\dot{G} - Y - k\dot{Y}$$

$$\ddot{Z} = -Z - Y + 2kY - kG + k$$

$$\begin{cases} \dot{Y} = -2Y + G - 1 \\ \dot{G} = Z \\ \dot{Z} = (2k-1)Y - kG - Z + k \end{cases}$$

$$\bar{z} = 0$$

$$G = 1 + 2\gamma$$

$$(\cancel{2k}-1)\gamma - \cancel{k} - \cancel{2k}\gamma + \cancel{k} = 0$$

$$-\gamma = 0 \quad \bar{\gamma} = 0 \quad \bar{G} = 1$$

$$\det \begin{bmatrix} -2-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ 2k-1 & -k & -1-\lambda \end{bmatrix} = -\lambda(\lambda+1)(\lambda+2) + 2k-1 - k(\lambda+2)$$

$$= -\lambda(\lambda^2 + 3\lambda + 2) + \cancel{2k}-1 - k\lambda - \cancel{2k}$$

$$= -\lambda^3 - 3\lambda^2 - 2\lambda - k\lambda - 1$$

$$\lambda^3 + 3\lambda^2 + \underbrace{(2+k)}_{>0}\lambda + 1$$

$$\left[\begin{array}{cc|c} 3 & 1 & 0 \\ 1 & 2+k & 0 \\ \hline 0 & 3 & 1 \end{array} \right]$$

$$3(2+k) > 1$$

$$2+k > \frac{1}{3} \quad k > -\frac{5}{3}$$

always
true.

$$5) \quad f(x) = x(x^2 - \alpha) \quad \alpha \in \mathbb{R}$$

$$x(x^2 - \alpha) = x \quad \overline{x} = 0$$

$$x^2 - \alpha = 1 \quad x^2 = 1 + \alpha \quad \alpha > -1$$

$$x = \pm \sqrt{1 + \alpha}$$

$$f'(x) = 3x^2 - \alpha$$

$$f'(0) = -\alpha \quad -1 < \alpha < 1 \quad \text{A.S.}$$

$$\alpha < -1 \vee \alpha > 1 \quad \text{UNST.}$$

$$f'(\pm \sqrt{1 + \alpha}) = 3 + 3\alpha - \alpha = 3 + 2\alpha$$

$$-1 < 3 + 2\alpha < 1 \quad -4 < 2\alpha < -2$$

$$-2 < \alpha < -1$$

$$\alpha > -1 \Rightarrow \pm \sqrt{1 + \alpha} \quad \text{UNST.}$$