

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **8th June 2026**

1. A gambler starts from 4€ and aims to reach a total of 15€. If the probability of winning is 0.45, what is the probability that the gambler gets ruined?

What happens if the probability of winning raises to 1/2? In this case, what would be the target N needed to have exactly a 60% chance of getting ruined?

2. Discuss the stability of equilibria of the linear second-order discrete equation

$$x_{h+2} - \frac{3}{2}x_{h+1} + \alpha x_h = 3$$

for $\alpha \in \mathbb{R}$.

In the case $\alpha = -1$, find the explicit solution for $x_0 = 0$, $x_1 = 0$.

3. The interaction between the price and the supply of a good follows the system of ODEs

$$\begin{cases} \dot{P} = P(2 - P - S) \\ \dot{S} = S(3 - 2P - S). \end{cases}$$

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -2Y + G - 1 \\ \dot{G} = -G - \int_0^t Y(\tau) d\tau - kY \end{cases}$$

for $k > 0$.

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h(x_h^2 - \alpha), \quad \alpha \in \mathbb{R}.$$